<u>Large Weapons From MATH 235</u>

1. *Def(n):* f is a function from the set X into the set Y ($f : X \to Y$) iff f assigns to <u>each</u> element in X a <u>unique</u> element of Y. The set X is then called the domain of f and the set Y is then called the range of f. With $D_f =$ domain of f and $R_f =$ range of f, we have $f = \{(x, y) \mid \forall x \in D_f \ \ \exists!\, y \in R_f, \textit{ which we denote as } y = f(x)\}$.

2. *Def(n):* $\lim_{x\to c} f(x) = L$ iff $\forall \varepsilon > 0 \ \exists \ \delta > 0 \ \ s.t. \ \ 0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

3. *Def(n):* f is continuous at $x = c$ iff $\lim_{x\to c} f(x) = f(c)$

4. *Th(m):* **EVT** (Extreme Value Theorem) If f is continuous on $[a, b]$ then

$$\exists \ c_1, c_2 \in [a,b] \ \ s.t. \ \ f(x) \in [\, f(c_1), f(c_2)\,] \ \forall\, x \in [a,b]$$

5. *Th(m):* **IVT** (Intermediate Value Theorem) If f is continuous on $[a, b]$ and $\exists \ k$ between $f(a)$ and $f(b)$ then $\exists\, c \in (a,b)$ $s.t.$ $f(c) = k$.

6. *Def(n):* f is **differentiable** at $x = c$ iff $\lim_{x\to c} \frac{f(x) - f(c)}{x - c}$ or $\lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$

Then we say $f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$ or equivalently, $f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$.

7. *Th(m):* f is **differentiable** at $x = c$ $\Rightarrow$ f is **continuous** at $x = c$

8. *Def(n):* f is a continuous or differentiable function on an interval I iff it is continuous or differentiable at each $c \in I$.

9. *Def(n):* $C^{(k)}(I)$ - is the set of functions whose first k derivatives are continuous on the interval I.

10. *Th(m):* "LIMIT BOY" **Composition :** If g is cont. at c and f is cont. at $g(c)$

$$\Rightarrow \lim_{x\to c} f(g(x)) = f(\lim_{x\to c} g(x)) = f(g(c))$$

11. *Th(m):* **Pinching Theorem**

$$\lim_{x\to c} h(x) = L \quad \& \quad \lim_{x\to c} g(x) = L \ \ \& \ \ h(x) \le f(x) \le g(x)$$

$$\Rightarrow \lim_{x\to c} f(x) = L$$

12. *Def(n):* f is **increasing** on an interval I iff $\forall\, x_1, x_2 \in I$ such that $x_1 < x_2 \;\Rightarrow\; f(x_1) < f(x_2)$

13. *Th(m):* **Rolle's Theorem** - If f is continuous on $[a,b]$ and f is differentiable on (a,b) and $f(a) = f(b) = 0$ then $\exists\, c \in (a,b)$ *s.t.* $f'(c) = 0$

14. *Th(m):* **MVT** (Mean Value Theorem) - If f is continuous on $[a,b]$ and f is differentiable on (a,b) then $\exists\, c \in (a,b)$ *s.t.* $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

15. *Th(m):* If $f'(x) > 0 \;\forall\, x \in I$, then f is **increasing** on the interval I .

16. *Th(m):* If $f'(x) = 0 \;\forall\, x \in I$, then f is **constant** on an interval I

17. *Def(n):* The number $\int_a^b f(x)dx$ is the unique number that equals the common value $\lim_{n\to\infty} U_f(n) = \lim_{n\to\infty} L_f(n)$ where $U_f(n), L_f(n)$ are Upper and Lower sums for *f* with *n* equally spaced subdivisions of $[a,b]$. Or more generally, $\int_a^b f(x)dx$ is the unique number that lies between all upper sums and lower sums of all partitions of $[a,b]$.

18. *Th(m):* The number $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$ where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$ for $i = 0, \cdots, n$.

19. *Th(m):* **Generalized MVTI Theorem -** If f, g are continuous on $[a,b]$ and $g(x) \ge 0 \;\forall\; x \in [a,b]$ then $\exists\, c \in [a,b]$ *s.t.* $\int_a^b f(x)g(x)dx \;=\; f(c)\int_a^b g(x)dx$.
Choosing $g(x) \equiv 1$ gives **MVTI**: f continuous on $[a,b] \Rightarrow$
$\exists\, c \in [a,b]$ *s.t.* $\int_a^b f(x)dx \;=\; f(c)(b-a)$, or $f(c) = \dfrac{1}{b-a}\int_a^b f(x)dx$.

20. *Th(m):* **First Fundamental Theorem of Calculus (FTCI)-** If G is an antiderivative of *f, that is,* If $\exists\, G \in C^1[a,b]$ *s.t.* $G'(x) = f(x) \;\forall\; x \in [a,b]$,
then $\int_a^b f(x)dx = G(x)\big|_a^b = G(b) - G(a)$.

21. *Th(m):* **Fundamental Theorem** II- If f is continuous on $[a,b]$ then $\forall x \in [a,b]$,

$\frac{d}{dx}\int_a^x f(t)dt = f(x)$. Equivalently, if $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x) \ \ \forall x \in [a,b]$.

22. *Def(n):* $\lim_{x\to\infty} f(x) = L$ iff $\forall \varepsilon > 0 \ \exists \ N > 0 \ s.t. \ x > N \Rightarrow |f(x) - L| < \varepsilon$

A preview of a few important M236 ideas to come:

1. A sequence $\{a_n\} \to A$ iff $\forall \ \varepsilon > 0 \ \exists \ N > 0 \ s.t. \ n > N \Rightarrow |a_n - A| < \varepsilon$.

A sequence of numbers $\{a_n\} = \{a_n\}_{n=1}^{\infty} = \{a_1, a_2, a_3, ...\}$ or similarly, $\{a_n\} = \{a_n\}_{n=0}^{\infty}$.

2. **Taylor's Theorem with Remainder** - If $f \in C^{k+1}[a,b]$ and $x, c \in (a,b)$ then $\exists \xi$ (which depends on x, so really $\xi(x)$) between c and x such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(k)}(c)}{k!}(x-c)^k + \frac{f^{(k+1)}(\xi)}{(k+1)!}(x-c)^{k+1}.$$

3. **Taylor's Theorem with Remainder** - If $f \in C^{k+1}[a,b]$ and $x, x+h \in (a,b)$ then $\exists \xi$ (which depends on x, so really $\xi(x)$) between $x+h$ and x such that

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \cdots + \frac{f^{(k)}(x)}{k!}h^k + \frac{f^{(k+1)}(\xi)}{(k+1)!}h^{k+1}$$