INTEGRAL TOOLS

Def(n): The number $\int_a^b f(x)dx$ is the unique number that lies between the upper sum and the lower sum of all partitions of $[a,b]$.

Def(n): The family of functions $\int f(x)\,dx$ is the set of ALL functions whose derivative is f.

Th(m): The number $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$ where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$ for $i = 0,\cdots,n$ for each n.

Th(m): **Fundamental Theorem** I- If $\exists\ F \in C[a,b]$ *s.t.* $F'(x) = f(x)\ \ \forall\ \ x \in [a,b]$ then $\int_a^b f(x)dx = F(x)\big|_{x=a}^{x=b} = F(b) - F(a)$.

Th(m): **Fundamental Theorem** II- If $F(x) = \int_a^x f(t)\,dt$ then $F'(x) = f(x)\ \forall\ x \in [a,b]$.

1. $\int [g(x)]^k\ g'(x)\,dx = \frac{[g(x)]^{k+1}}{k+1} + C$ where k is rational

2. $\int \sin(g(x))g'(x)\,dx = -\cos(g(x)) + C$

3. $\int \cos(g(x))g'(x)\,dx = \sin(g(x)) + C$

4. $\int \sec^2(g(x))g'(x)\,dx = \tan(g(x)) + C$

5. $\int \csc^2(g(x))g'(x)\,dx = -\cot(g(x)) + C$

6. $$\int \sec(g(x))\tan(g(x))g'(x)\,dx = \sec(g(x)) + C$$

7. $$\int \csc(g(x))\cot(g(x))g'(x)\,dx = -\csc(g(x)) + C$$

8. $\int_a^b [g(x)]^k g'(x)\,dx = \frac{[g(x)]^{k+1}}{k+1}\Bigg|_a^b$ where k is rational

9. $\int_a^b \sin(g(x))g'(x)\,dx = -\cos(g(x))\Big|_a^b$

10. $\int_a^b \cos(g(x))g'(x)\,dx = \sin(g(x))\Big|_a^b$

11. $\int_a^b \sec^2(g(x))g'(x)\,dx = \tan(g(x))\Big|_a^b$

12. $\int_a^b \csc^2(g(x))g'(x)\,dx = -\cot(g(x))\Big|_a^b$

13. $$\int_a^b \sec(g(x))\tan(g(x))g'(x)\,dx = \sec(g(x))\Big|_a^b$$

14. $$\int_a^b \csc(g(x))\cot(g(x))g'(x)\,dx = -\csc(g(x))\Big|_a^b$$