LIMIT TOOLS

LT1. $\lim_{x\to c}(mx+b) = mc+b$

LT2. $\lim_{x\to c} f(x) = L$
$\Rightarrow \lim_{x\to c}(kf(x)) = k\lim_{x\to c} f(x)$

LT3. $\lim_{x\to c} f(x) = L \;\&\; \lim_{x\to c} g(x) = M$
$\Rightarrow \lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = L+M$

LT4. $\lim_{x\to c} f(x) = L \;\&\; \lim_{x\to c} g(x) = M$
$\Rightarrow \lim_{x\to c}(f(x)*g(x)) = \lim_{x\to c} f(x) * \lim_{x\to c} g(x) = L*M$

LT5. $\lim_{x\to c} f(x) = L \;\&\; \lim_{x\to c} g(x) = M \neq 0$
$\Rightarrow \lim_{x\to c}(f(x)/g(x)) = \lim_{x\to c} f(x) / \lim_{x\to c} g(x) = L/M$

LT6. $\lim_{x\to c} f(x) = L \;\&\; \lim_{x\to c} f(x) = M \Rightarrow L = M$

LT7. $\lim_{x\to c} f(x) = L \neq 0 \;\;\&\;\; \lim_{x\to c} g(x) = 0$
$\Rightarrow \lim_{x\to c} \frac{f(x)}{g(x)} = D.N.E.$

LT8. $P(x)$ is polynomial means
$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$
$P_1(x)$ & $P_2(x)$ are polynomials & $P_2(c) \neq 0$
then $\lim_{x\to c} \frac{P_1(x)}{P_2(x)} = \frac{P_1(c)}{P_2(c)}$

LT9. $\left.\begin{array}{l} \lim_{x\to c} f(x) = L \;\;\& \\ f(x) = g(x) \;\; \forall x \in (c-\delta, c)\cup(c, c+\delta) \\ \text{for some } \delta > 0 \end{array}\right\} \Rightarrow \lim_{x\to c} g(x) = L$

LT10. $\lim_{x\to c} f(x) = L \Leftrightarrow \lim_{x\to c^+} f(x) = L \;\; AND \;\; \lim_{x\to c^-} f(x) = L$

LT11. $\lim_{x\to\pm\infty} \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0} = \lim_{x\to\pm\infty} \frac{a_n x^n}{b_m x^m}$

LT12. $\lim_{x\to c} f(x) = L \Rightarrow \lim_{x\to c}(|f(x)|) = |L| \quad (L = 0 \Leftrightarrow)$

LIMIT BOY

LT13. g is cont. at c & f is cont. at $g(c)$
$\Rightarrow \lim_{x\to c} f(g(x)) = f(\lim_{x\to c} g(x)) = f(g(c))$

also true $\lim_{x\to c} g(x) = L$ & f is cont. at L
$\Rightarrow \lim_{x\to c} f(g(x)) = f(\lim_{x\to c} g(x)) = f(L)$

VALID FOR $(x\to c^+, x\to c^-, x\to\infty, x\to-\infty)$

PINCHING THEOREM

LT14.
$\lim_{x\to c} f(x) = L \;\;\&\;\; \lim_{x\to c} g(x) = L \;\;\&\;\; f(x) \le h(x) \le g(x)$
$\Rightarrow \lim_{x\to c} h(x) = L$

LT15. $\lim_{x\to c}\sin(x) = \sin(c) \quad \& \quad \lim_{x\to c}\cos(x) = \cos(c)$

LT16. $\lim_{x\to 0}\frac{\sin(kx)}{kx} = 1$

LT17. $\lim_{x\to 0}\frac{1-\cos(kx)}{kx} = 0$

LT18. $\lim_{x\to c} f(x) = f(c) \Leftrightarrow \lim_{x\to 0} f(x+c) = f(c)$

LT19. $\lim_{x\to 0} f(x) = L \Leftrightarrow \lim_{x\to 0} f(kx) = L$