

**For the following, find where a function is increasing or decreasing. Then find where it takes on local or global maximum or minimum values if these values exist.**

2. The function here is given as  $f(x) = \frac{2x}{x+3}$ .

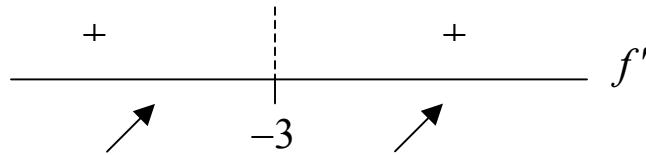
- i. Determine the **domain** of  $f$ . Since with  $\frac{2x}{x+3}$  there is a fraction and  $x = -3$  will result in division by 0 we must eliminate  $x = -3$  from the domain. There is no possibility taking the radical of a number less than 0, so the domain for  $f$  is all real numbers,  $(-\infty, -3)$  and  $(-3, \infty)$ . Next find

the derivative of  $f$ . For  $f(x) = \frac{2x}{x+3}$  using quotient rule,

$$\begin{aligned} f'(x) &= \frac{(2x)'(x+3) - (2x)(x+3)'}{(x+3)^2} \\ &= \frac{2(x+3) - (2x)1}{(x+3)^2} \\ &= \frac{6}{(x+3)^2} \end{aligned}$$

- ii. Determine all possible **critical points** for the function. Recall critical points are values in the domain of  $f$  where the derivative  $f'$  is 0, or does not exist. Since  $\frac{6}{(x+3)^2}$  only doesn't exist at  $x = -3$ , and this value **isn't** in the domain of  $f$ ,  $x = -3$  **IS NOT** a critical point. So  $f'$  always exist on  $(-\infty, -3)$  and  $(-3, \infty)$ . Critical points will only possibly then occur at values of  $x$  where  $f'(x) = 0$ . Setting  $f'(x) = 0$  we get that  $\frac{6}{(x+3)^2} = 0$ . Recall from algebra that a fraction can only be 0 when the numerator is 0, but  $6 \neq 0$  so  $f'(x) \neq 0$  on  $(-\infty, -3)$  and  $(-3, \infty)$ . We reach the conclusion that there aren't any critical points.

- iii. Next we create a  $\pm$  number line **sign chart** for  $f'$ . First mark the point  $-3$  on the sign chart as it was excluded from the domain. Since with  $f'(x) = \frac{6}{(x+3)^2}$  the numerator and denominator are always + the derivative is always +. We can now easily create the following sign chart.



- iv. Using the sign chart for the first derivative we can easily find intervals of **increase** and **decrease**. As  $f' > 0$  on both sides of  $x = -3$ , the original function  $f$  is increasing when  $x$  is less than  $-3$  and when  $x$  is greater than  $-3$ . As there are no critical points there are no **LOCAL MAX's** or **Min's**.
- v. Since a **GLOBAL MAX** will occur either at an end point of the domain of  $f$  or at a LOCAL MAX we can see that there is NO GLOBAL MAX, as there are no end points in  $(-\infty, -3)$  and  $(-3, \infty)$ . Similarly there isn't a GLOBAL MIN.

### Summary:

- 1) Critical Points: NONE
- 2) Intervals of Increase:  $(-\infty, 3)$  and  $(-3, \infty)$
- 3) Intervals of Decrease: NONE
- 4) Local Max's: NONE
- 5) Local Min's: NONE
- 6) Global Max: NONE
- 7) Global Min: NONE