Math/Phys 266E
Homework #3
(This looks longer than it is!)

Assigned: Tu Feb. 27 *Due: W March 14*

1. For the motion in Spencer, p.42, #1: Compute the matrix form of $\dfrac{D\mathbf{F}(\mathbf{X},t)}{Dt}$ by:

(a) first computing the deformation gradient tensor $\mathbf{F}$ and then directly computing $\dot{\mathbf{F}}$.
(b) first computing $\mathbf{v}(\mathbf{X},t)$ and then computing the material velocity gradient, Grad**v.**
(c) What is true about (a) and (b)?

2. For the velocity field in Spencer, p.42, #4: Compute the component matrix of:

(a) $\mathbf{L}(\mathbf{x},t)$, the spatial velocity gradient tensor.
(b) $\mathbf{D}(\mathbf{x},t)$, the rate of deformation tensor.
(c) $\mathbf{W}(\mathbf{x},t)$, the spin tensor. Are there any values of t and any points $\mathbf{x}$ for which $\mathbf{W}$ is an orthogonal tensor? Justify.
(d) Determine the axial vector $\mathbf{w}(\mathbf{x},t)$ corresponding to $\mathbf{W}$.

3. For the velocity field in Spencer, p. 43, #5, with $a_1 = A,\ a_2 = 0,\ b_1 = 0,\ b_2 = 2A,\ c = 3A$: Compute:

(a) The component matrix of $\mathbf{L}(\mathbf{x},t)$, the spatial velocity gradient tensor.
(b) The particle paths (i.e., the motion $\mathbf{x} = \mathbf{x}(\mathbf{X},t)$). Recall, at $t = 0,\ \mathbf{x} = \mathbf{X}$. *Since you can't do the rest without having this part correct, show that* $x_1 = X_1(1+t)^A, x_1 = X_2(1+t)^{2A}, x_3 = X_3(1+t)^{3A}$.
(c) The component matrix for the deformation gradient tensor $\mathbf{F}$. Also compute $J = \det(\mathbf{F})$.
(d) The component matrix for the material time derivative of $\mathbf{F}$ (i.e., $\dot{\mathbf{F}}$).
(e) Compare $\dot{\mathbf{F}}$ and $\mathbf{LF}$.
(f) Compute $\mathbf{F}^{-1}$ and compare $\mathbf{L}$ and $\dot{\mathbf{F}}\mathbf{F}^{-1}$.
(g) The left and right Cauchy-Green deformation tensors $\mathbf{B}$ and $\mathbf{C}$, respectively.
(h) The Lagrangian and Eulerian strain tensors $\boldsymbol{\gamma}$ and $\boldsymbol{\eta}$, respectively.
(i) Suppose that the original configuration of the body undergoing this motion was a unit cube defined for $0 \le X_i \le 1,\ i = 1,2,3$, with volume defined by $\mathbf{E}_1 \bullet (\mathbf{E}_2 \times \mathbf{E}_3)$. What are the dimenions (i.e., where do the x_i live) and volume of the deformed body at time $t = 1$ if $A = 1$?

4. Consider the motion in Example 2.2 in Holzapfel, p.63. Compute the matrices of:

(a) The deformation gradient tensor $\mathbf{F}$. Now compute, the Jacobian determinant $J = \det(\mathbf{F})$. For what *t*, if any, is this an isochoric motion?
(b) The left and right Cauchy-Green deformation tensors $\mathbf{B}$ and $\mathbf{C}$, respectively.
(c) The Lagrangian and Eulerian strain tensors $\boldsymbol{\gamma}$ and $\boldsymbol{\eta}$, respectively.
(d) The infinitesimal strain tensor $\mathbf{E}$. *Hint:* Find an exact formula to compute $\mathbf{E}$ *without* computing the displacement gradient tensor!