Math/Phys 266E
Homework #2

Assigned: Tu Jan. 30 *Due: Th Feb. 22*

1. Prove each of the following **in direct notation** and indicate whether each is a tensor, a vector, or a scalar.

a). $(\mathbf{a}+\mathbf{b})\otimes\mathbf{c}=\mathbf{a}\otimes\mathbf{c}+\mathbf{b}\otimes\mathbf{c}$ *Tensor, vector, scalar* ? (circle one)

b). $\mathbf{a}\otimes(\mathbf{b}+\mathbf{c})=\mathbf{a}\otimes\mathbf{b}+\mathbf{a}\otimes\mathbf{c}$ *Tensor, vector, scalar* ? (circle one)

c). $(\mathbf{a}\otimes\mathbf{b})\mathbf{v}=(\mathbf{a}\otimes\mathbf{v})\mathbf{b}$ *Tensor, vector, scalar* ? (circle one)

d). $(\mathbf{a}\otimes\mathbf{b})(\mathbf{c}\otimes\mathbf{d})=(\mathbf{b}\bullet\mathbf{c})(\mathbf{a}\otimes\mathbf{d})$ *Tensor, vector, scalar* ? (circle one)

e). $(\mathbf{e}_i\otimes\mathbf{e}_i)(\mathbf{e}_j\otimes\mathbf{e}_j)=\begin{cases}\mathbf{0} & \text{if } i\neq j\\ (\mathbf{e}_i\otimes\mathbf{e}_i) & \text{if } i=j\end{cases}$ *Tensor, vector, scalar* ? (circle one)

f). $\mathbf{T}(\mathbf{a}\otimes\mathbf{b})=(\mathbf{T}\mathbf{a})\otimes\mathbf{b}$ *Tensor, vector, scalar* ? (circle one)

g). $(\mathbf{T}\mathbf{e}_i)\otimes\mathbf{e}_i=\mathbf{T}$ *Tensor, vector, scalar* ? (circle one)

h). $tr(\mathbf{I})=$ _____? *Tensor, vector, scalar* ? (circle one)
Fill in the blank **and** prove.

2. Is the tensor product of two vectors a *commutative* product? Justify your answer without appealing to components.

3. For each of the following, the action of $\mathbf{T}$ is as defined below where $\mathbf{T}:\mathbb{R}^3\to\mathbb{R}^3$, $\mathbf{u}$ is an arbitrary vector, $\mathbf{n}$ is a *fixed, given* vector, and let *k* is an arbitrary scalar. Determine whether $\mathbf{T}$ is a tensor in each case below.

(a) $\mathbf{T}\mathbf{u}=\mathbf{n}$

(b) $\mathbf{T}\mathbf{u}=k\mathbf{u}$

(c) $\mathbf{T}\mathbf{u}=(\mathbf{u}\bullet\mathbf{u})\mathbf{u}$