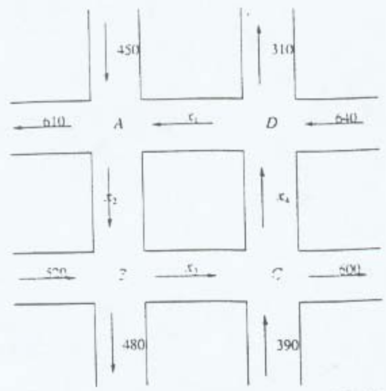
APPLICATION 1: TRAFFIC FLOW

In the downtown section of a certain city two sets of one-way streets intersect as shown in Figure 1.2.2. The average hourly volume of traffic entering and leaving this

FIGURE 1.2.2



section during rush hour is given in the diagram. Determine the amount of traffic between each of the four intersections.

SOLUTION. At each intersection the number of automobiles entering must be the same as the number leaving. For example, at intersection A, the number of automobiles entering is $x_1 + 450$ and the number leaving is $x_2 + 610$. Thus

$$x_1 + 450 = x_2 + 610 \qquad \text{(intersection } A\text{)}$$

Similarly,

$$\begin{aligned} x_2 + 520 &= x_3 + 480 \qquad \text{(intersection } B\text{)} \\ x_3 + 390 &= x_4 + 600 \qquad \text{(intersection } C\text{)} \\ x_4 + 640 &= x_1 + 310 \qquad \text{(intersection } D\text{)} \end{aligned}$$

The augmented matrix for the system is

$$\left(\begin{array}{rrrr|r} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & -1 & 0 & -40 \\ 0 & 0 & 1 & -1 & 210 \\ -1 & 0 & 0 & 1 & -330 \end{array}\right)$$

The reduced row echelon form for this matrix is

$$\left(\begin{array}{rrrr|r} 1 & 0 & 0 & -1 & 330 \\ 0 & 1 & 0 & -1 & 170 \\ 0 & 0 & 1 & -1 & 210 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

The system is consistent and since there is a free variable, there are many possible solutions. The traffic flow diagram does not give enough information to determine x_1, x_2, x_3, x_4 uniquely. If the amount of traffic were known between any pair of intersections, the traffic on the remaining arteries could easily be calculated. For example, if the amount of traffic between intersections C and D averages 200 automobiles per hour, then $x_4 = 200$. One can then solve for x_1, x_2, x_3 in terms of x_4.

$$\begin{aligned} x_1 &= x_4 + 330 = 530 \\ x_2 &= x_4 + 170 = 370 \\ x_3 &= x_4 + 210 = 410 \end{aligned}$$ ◄