HW QUIZ #1 score___/35 **Name** Key

Math 238: 2.1, 2.2

JUSTIFY EVERYTHING!!!!!!

12 pts 1. a) Determine whether the *Commutative Property of Addition* for a vector space hold for the addition and scalar multiplication on ordered pairs of real numbers defined by

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1, y_2)$$
$$c \odot (x, y) = (cx, cy)$$

(4) No. Let $x_1, y_1, x_2, y_2 \in \mathbb{R}$

Then $(x_1,y_1) \oplus (x_2,y_2) \overset{\oplus}{=} (x_1,y_2) \neq (x_2,y_1) \overset{\oplus}{=} (x_2,y_2) \oplus (x_1,y_1)$

(4) b) Determine whether each of the two *Distributive Law Properties* for a vector space hold under the above definitions.

$c(u+v) = cu+cv \quad \forall c \in \mathbb{R}, u,v \in V$

Let $c \in \mathbb{R}$, $x_1, x_2, y_1, y_2 \in \mathbb{R}$

$c \odot ((x_1,y_1) \oplus (x_2,y_2)) \overset{\oplus}{=} c \odot (x_1, y_2)$
$\overset{\odot}{=} (cx_1, cy_2)$ (LHS)

$cu + cv = (c \odot (x_1,y_1)) \oplus (c \odot (x_2,y_2))$
$\overset{\odot}{=} (cx_1, cy_1) \oplus (cx_2, cy_2) \overset{\oplus}{=} (cx_1, cy_2)$ (RHS)

so LHS = RHS $\rightarrow$ YES!

(4) $(c+d)u = cu + du \quad \forall c,d \in \mathbb{R}, u \in V$

Let $c, d \in \mathbb{R}$, $x, y \in \mathbb{R}$. Then

$(c+d) \odot (x,y) \overset{\odot}{=} ((c+d)x, (c+d)y)$
$= (cx+dx, cy+dy)$ Distr. Prop. for $\mathbb{R}$ (LHS)

$c(x,y) \oplus d(x,y) \overset{\odot}{=} (cx, cy) \oplus (dx, dy)$
$\overset{\oplus}{=} (cx, dy) \rightarrow$ RHS
$\neq (cx+dx, cy+dy)$

So NO! LHS $\neq$ RHS.

8 pts 2. Determine the *zero vector and additive inverse vector* for the following vector space: The set of ordered pairs of positive real numbers under the addition and scalar multiplication defined by the following:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 x_2, y_1 y_2)$$
$$c \odot (x, y) = (x^c, y^c)$$

(4) *Zero vector:* Let $x, y, z_1, z_2 \in \mathbb{R}^+$

Want $(x, y) \oplus (z_1, z_2) = (x, y)$

By $\oplus$ $(x,y) \oplus (z_1, z_2) = (x z_1, y z_2)$

so $(x z_1, y z_2) = (x, y) \Rightarrow$

$\boxed{(z_1, z_2) = (1, 1) = \underset{\sim}{0} \text{ vector}}$

(4) *Additive Inverse vector:* Let $x, y \in \mathbb{R}^+$; $z_1, z_2 \in \mathbb{R}$

Want $(x,y) \oplus (z_1, z_2) = (1,1) \leftarrow$ zero vector

By $\oplus$ $(x,y) \oplus (z_1, z_2) = (x z_1, y z_2)$

so $x z_1 = 1$ + $y z_2 = 1 \Rightarrow z_1 = \frac{1}{x}, z_2 = \frac{1}{y}$

$\boxed{\text{So additive inverse} = \left(\frac{1}{x}, \frac{1}{y}\right), \text{ o.k. since } x, y > 0}$

5 pts 3. Is the set of all vectors of the form (x, y, x^2+y^2), where $x, y \in \mathbb{R}$, a subspace of $\mathbb{R}^3$? No. Not closed under $\oplus$ since for $x_1, y_1, x_2, y_2 \in \mathbb{R}$

$(x_1, y_1, x_1^2+y_1^2) \oplus (x_2, y_2, x_2^2+y_2^2) = (x_1+x_2, y_1+y_2, x_1^2+y_1^2+x_2^2+y_2^2)$
$\neq (x_1+x_2, y_1+y_2, (x_1+x_2)^2 + (y_1+y_2)^2)$
$x_1^2 + 2x_1x_2 + x_2^2 + y_1^2 + 2y_1y_2 + y$

10 pts 4. Is the set $I \equiv \{f \in C[a,b] \text{ for which } \int_a^b f(x)dx = 0\}$ a subspace of $F[a,b]$? Yes. I is closed under $\oplus$ and $\odot$!

(5) $\oplus$ Let $f, g \in I$. Then $f, g \in C[a,b]$ + $\int_a^b f(x)dx = 0$, $\int_a^b g(x)dx = 0$

Consider $f+g$. $f+g \in C[a,b]$ since $C[a,b]$ is a vector space (OR sum of cont. fns is cont.)

Also, $\int_a^b (f+g)(x)dx = \int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx = 0 + 0$ since $f, g \in I$ $= 0$ so $f+g \in I$.

(5) $\odot$ Let $c \in \mathbb{R}$, $f \in I$. Then $(cf) \in C[a,b]$ since $C[a,b]$ is a vector space (OR scalar mult of cont. fn. is continuous.)

Also $\int_a^b (cf)(x)dx = \int_a^b cf(x)dx = c\int_a^b f(x)dx \underset{\text{since } f \in I}{=} c \cdot 0 = 0 \Rightarrow cf \in I$