Type	$g(x)$	$y_p(x)$
(I)	$p_n(x) = a_n x^n + \cdots + a_1 x + a_0$	$x^s P_n(x) = x^s\{A_n x^n + \cdots + A_1 x + A_0\}^\dagger$
(II)	$ae^{\alpha x}$	$x^s A e^{\alpha x}$
(III)	$a \cos \beta x + b \sin \beta x$	$x^s \{A \cos \beta x + B \sin \beta x\}$
(IV)	$p_n(x) e^{\alpha x}$	$x^s P_n(x) e^{\alpha x}$
(V)	$p_n(x) \cos \beta x + q_m(x) \sin \beta x$, where $q_m(x) = b_m x^m + \cdots + b_1 x + b_0$	$x^s \{P_N(x) \cos \beta x + Q_N(x) \sin \beta x\}$, where $Q_N(x) = B_N x^N + \cdots + B_1 x + B_0$ and $N = \max(n, m)$
(VI)	$ae^{\alpha x} \cos \beta x + be^{\alpha x} \sin \beta x$	$x^s \{Ae^{\alpha x} \cos \beta x + Be^{\alpha x} \sin \beta x\}$
(VII)	$p_n(x) e^{\alpha x} \cos \beta x + q_m(x) e^{\alpha x} \sin \beta x$	$x^s e^{\alpha x}\{P_N(x) \cos \beta x + Q_N(x) \sin \beta x\}$, where $N = \max(n, m)$

The nonnegative integer s is chosen to be the smallest integer so that no term in the particular solution $y_p(x)$ is a solution to the corresponding homogeneous equation $L[y](x) = 0$.

† $P_n(x)$ must include all its terms even if $p_n(x)$ has some terms that are zero.

Euler Eqns:

$$x^2 y'' + Axy' + By = 0, \quad x > 0$$

$$z = \ln x \rightarrow \frac{d^2y}{dz^2} + (A-1)\frac{dy}{dz} + By = 0$$

LINEAR FIRST ORDER EQUATIONS

A general solution to the first order linear equation $dy/dx + P(x)y = Q(x)$ is

$$y(x) = [\mu(x)]^{-1}\left(\int \mu(x) Q(x)\,dx + C\right), \quad \text{where} \quad \mu(x) = \exp\left(\int P(x)\,dx\right).$$

REDUCTION OF ORDER FORMULA

Given a nontrivial solution $f(x)$ to $y'' + py' + qy = 0$, a second linearly independent solution is

$$y(x) = f(x) \int \frac{e^{-\int p(x)\,dx}}{[f(x)]^2}\,dx.$$

VARIATION OF PARAMETERS FORMULA

If y_1 and y_2 are two linearly independent solutions to $y'' + py' + qy = 0$, then a particular solution to $y'' + py' + qy = g$ is $y = v_1 y_1 + v_2 y_2$, where

$$v_1(x) = \int \frac{-g(x) y_2(x)}{W[y_1, y_2](x)}\,dx, \qquad v_2(x) = \int \frac{g(x) y_1(x)}{W[y_1, y_2](x)}\,dx,$$

and $W[y_1, y_2](x) = y_1(x) y_2'(x) - y_1'(x) y_2(x)$.