You are hired to housesit for your rich uncle, who asks you to fill his 30,000 gallon capacity indoor pool with a chlorine and water mixture using his state of the art, variable input/outtake system that also keeps the chlorine and water solution well stirred. Preoccupied with reading your ODE book, you accidentally cause the mixture to enter at a rate of $3(t+1)^2$ gal./hr and leave at a rate of $2(t+1)^2$ gal./hr. There is $\frac{1}{2}$ kg of chlorine per gallon in the input mixture and the pool has $\frac{1}{3}$ gal. of pure water in it when you start the system.

a) Determine the volume of the chlorine and water mixture in the pool at any time t. When, if ever, will the pool overflow?

b) Set up an initial value problem and solve it to determine the amount of chlorine in the pool at any time t.

c) What is the concentration of chlorine at any time t?

Correction to IVP for part b): (That's what I get for trying to squeeze just one more thing in☺!)

The initial condition for the **amount** *of substance* at time t is **x(0) = 0** because the problem states that the pool initially has 1/3 gal. of **pure water, i.e., the amount of chlorine** *(chlorine is our substance!!)* at time t=0 is zero! Note that *C_i=1/2 kg chlorine/gallon is the initial concentration of the mixture flowing into the pool, but the initial concentration of the "mixture" in the pool at the start is zero, since it is pure water.* In an equation, that would be $\mathbf{C(0) = \frac{x(0)}{V(0)} = \frac{0}{1/3} = 0}$.

So,

$x(t) = \frac{1}{6}(t+1)^3 + \frac{C}{(t+1)^6}$ as we derived in class, but

$$x(0) = 0 \Rightarrow \frac{1}{6}(0+1)^3 + \frac{C}{(0+1)^6} = 0$$

$$or \quad \frac{1}{6} + C = 0 \Rightarrow C = -\frac{1}{6}$$

So our final solution to the problem for part b) is:

$$x(t) = \frac{1}{6}(t+1)^3 - \frac{1}{6}\frac{1}{(t+1)^6}$$

and then the answer to part c) is:

$$C(t) = \frac{x(t)}{V(t)} = \frac{\frac{1}{6}(t+1)^3 - \frac{1}{6}\frac{1}{(t+1)^6}}{\frac{1}{3}(t+1)^3} = \frac{1}{2} - \frac{1}{2}\frac{1}{(t+1)^9}.$$