TEST #3 **score____/100** **Name**____

Math 236 *Extra Credit:* What was the name of Fred Flintstone's boss?

Spring 08

1. Fill in the blanks in the following definitions:

6 points

A) A sequence is a function whose domain is the set of ____________________ numbers and whose range is the set of

______________________ numbers.

B) The $\lim_{n\to\infty} a_n = L$ if and only if for every $\varepsilon > 0$ there exist an $N > 0$ such that

if _________________ then ____.

2. Use the above definition to give a proper **PROOF** that the sequence $\{a_n\} = \left\{\frac{1}{\sqrt{n}}\right\}$ converges to 0.

8 points

PROOF

SCRAP

3. Determine whether the following limits exists and, if so, find the value of the limit. Provide intermediate mathematical justification.

12 points

A) $\lim_{x\to 0^+} x^x$.

3A) Answer ______________

B) $\lim_{x\to 0} \frac{e^x - 1}{x(1+x)}$

3B) Answer ______________

4. Circle the sequence(s) below that are strictly monotonic:

6 points

A) $\left\{\frac{\ln n}{n}\right\}$

B) $\left\{\frac{(n+2)!}{3^n}\right\}$

5. Answer "*TRUE*" OR "*FALSE*" (ENTIRE WORD) **NO NEED TO SHOW WORK** **6 points**

A) $\sum_{k=1}^{\infty} \frac{1}{k}$ converges. __________ B) $\lim_{n\to\infty} \frac{\ln(n)}{n^p}$ with $p > 0$ converges. __________ C) $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ converges. __________

6. What number does $\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+2} \right)$ converge to? (YOU MUST SHOW PRECISE WORK) **6 points**

6) Answer ______________________________

7. **Justify** whether the following series converges or diverges. $\sum_{k=1}^{\infty} \left(\frac{4k}{7k-1} \right)^k$. **8 points**

8. **Justify** whether the following series converges or diverges. $\sum_{k=1}^{\infty} \frac{\sin^2(k)}{2k^2-1}$. **8 points**

8) Answer ______________________________

12 points

9. Answer "*TRUE*" OR "*FALSE*" (ENTIRE WORD) **DON'T SHOW WORK**

A) $\lim_{k\to\infty} \frac{k^k}{k!}$ diverges __________ B) $\sum_{k=1}^{\infty} \frac{3}{10^k}$ converges to $\frac{10}{3}$ __________ C) $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ converges to 1 __________

D) $\sum_{k=0}^{\infty} (-1)^k \frac{\sqrt{k}}{k^2+1}$ converges absolutely __________ E) $\sum_{k=1}^{\infty} \frac{e^k}{\ln(k)}$ diverges __________ F) $\lim_{x\to\infty} \frac{x^{10000}}{e^x} = 0$ __________

10. Determine whether the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$ converges or diverges. Show your work. **10 points**

11. Let $\{a_n\}$ be a sequence that converges to L and let $\{b_n\}$ be the sequence such that $b_n = ka_n$, $(k \neq 0)$. Prove that $\{b_n\}$ converges to kL. That is, **Prove**: If $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} b_n = kL$. **8 points**

12. A) Justify that the following is a convergent series and determine whether convergence is absolute or conditional.

$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k+1}}$ **10 points**

Type of convergence: ___________________

B) Determine the smallest integer n such that the n^{th} partial sum will approximate the series with $|\text{error}| < 0.001$.

n = _______________