

1. Answer the following with a, b assumed positive.

11 pts.

A) $\ln(ab) =$ _____ B) $\ln(1) =$ _____? C) $\ln\left(\frac{1}{a}\right) =$ _____? D) $\ln'(1) =$ _____?

E) Justify using some of the results (A)-(C) that $\ln\left(\frac{a}{b}\right) =$ _____?

2. A) Justify that $f(x) = \ln(x)$ is invertible for $x > 0$.

6 pts.

B) Given part A), we defined this inverse function to be e^x . Use implicit differentiation to justify that $(e^x)' = e^x \forall x \in \mathbb{R}$.

3. Complete:

9 pts.

- A) The function $\ln(x)$ is defined such that the real number, $\ln(x) =$ _____ where $x \in$ _____.
B) The function π^x is defined such that the real number, $\pi^x =$ _____ where $x \in$ _____.
C) The function x^π is defined such that the real number, $x^\pi =$ _____ where $x \in$ _____.

4. Find the derivative of the following functions and circle the appropriate domain for $f'(x)$ among the choices given:

15 pts.

A) $f(x) = \tan^{-1}(2^x)$

Domain of $f'(x)$?:

- $x \in \mathbb{R}$
or
 $x \in (0, \infty)$
or
 $x \in [0, \infty)$

B) $f(x) = e^{\sin^{-1}(x)}$

Domain of $f'(x)$?:

- $x \in \mathbb{R}$
or
 $x \in [-1, 1]$
or
 $x \in (-1, 1)$

C) $f(x) = \sqrt{\log_3(x)}$

Domain of $f'(x)$?:

- $x \in [1, \infty)$
or
 $x \in (0, \infty)$
or
 $x \in (1, \infty)$

5. Evaluate the following integrals:

20 pts. ²

A) $\int \frac{1}{x\sqrt{x^2-1}} dx$

B) $\int \cot x dx$

C) $\int 2^x + x^2 dx$

D) $\int \frac{1}{x(9 + [\ln(x)]^2)} dx$

E) $\int \frac{\sin(1 + e^{-x})}{e^x} dx$

6. Let $f(x) = \cos(x)$ then answer the following:

15 pts.

A) $\cos(\cos^{-1}(x)) = \underline{\hspace{2cm}}$ for all $x \in \underline{\hspace{2cm}}$

B) $\cos^{-1}(\cos(x)) = \underline{\hspace{2cm}}$ for all $x \in \underline{\hspace{2cm}}$

C) $\cos^{-1}(0) = ?$ circle one of : 0 1 -1 $\frac{\pi}{2}$ π DNE
circle one of : 0 1 -1 $\frac{\pi}{2}$ π DNE

D) $\cos^{-1}(\pi) = ?$

E) $\cos^{-1}(\cos(3\pi)) = \underline{\hspace{2cm}}$

F) $\tan(\cos^{-1}(x)) = \underline{\hspace{2cm}}$

7. Find the derivative of $f(x) = x^{\tan(x)}$ for $x \in (0, \pi/2)$.

5 pts.

8. Prove: $\lim_{x \rightarrow \infty} e^x \rightarrow \infty$. Hint: $\lim_{x \rightarrow \infty} f(x) \rightarrow \infty$ iff $\forall M > 0 \exists N > 0$ s.t. $x > N \Rightarrow f(x) > M$. 7 pts.

Give a proper direct proof write-up, supplying reasons for your steps—you may use properties of $\ln(x)$ in your proof!

SCRAP WORK:

9. Answer the following as TRUE or FALSE:

6 pts.

A) The inverse function of π^x is $x \ln(\pi)$.

A) _____

B) The following is true for all a and b , $e^{a-b} = \frac{\ln(\pi) \left[\log_{\pi} (e^a) \right]}{\ln(e^{e^b})}$.

B) _____

C) If $f(x) = 3x^3 - 5$ then $f^{-1}(x) = \sqrt[3]{3x-5}$.

C) _____

10. Suppose that f is a differentiable function such that the following identity holds: $\int_0^x f(t) dt = e^{2x} - \int_0^x e^{-t} f(t) dt$.

6 pts.

Determine $(f^{-1})'(1)$.