Quiz #4 score______/30 **Name** Key
Math 236
Spring 2008

SHOW WORK For Credit !!!!

1. Determine if the series converges or diverges. If it converges, find its sum. $\sum_{k=1}^{\infty}\left(-\frac{3}{2}\right)^k$ 10 pts.

Geometric Series.

$x=-\frac{3}{2}$ $|x|=\frac{3}{2}>1$

$\therefore$ Series diverges!

OR
$a_k=\left(-\frac{3}{2}\right)^k \to \pm\infty$ as $k\to\infty$
$\lim_{k\to\infty} a_k = \lim_{k\to\infty}\left(-\frac{3}{2}\right)^k \not\to 0$
so Diverges by Div. Test.

Answer: $\boxed{\text{Diverges}}$

2. Determine if the series converges or diverges. If it converges, find its sum. $\sum_{k=1}^{\infty}\left(\frac{1}{(k+2)(k+3)}\right)$ 10 pts.

Telescoping Series

so Look at $\{S_n\}_{n=1}^{\infty}$:

$S_1 = a_1 = \frac{1}{3}-\frac{1}{4}$

$S_2 = S_1 + a_2 = \left(\frac{1}{3}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{4}}-\frac{1}{5}\right)$

$S_3 = S_2 + a_3 = \frac{1}{3}-\cancel{\frac{1}{5}}+\left(\cancel{\frac{1}{5}}-\frac{1}{6}\right)$

$S_4 = S_3 + a_4 = \frac{1}{3}-\cancel{\frac{1}{6}}+\left(\cancel{\frac{1}{6}}-\frac{1}{7}\right)$

$\vdots$

$S_n = \frac{1}{3}-\frac{1}{n+3}$ and $\lim_{n\to\infty} S_n = \frac{1}{3}-0$

$a_k = \frac{1}{(k+2)(k+3)} = \frac{A}{(k+2)}+\frac{B}{(k+3)} = \frac{1}{k+2}-\frac{1}{k+3}$

$\Rightarrow 1 = A(k+3)+B(k+2)$

$k=-2 \Rightarrow \boxed{1=A}$ $k=-3 \Rightarrow 1=-B$
$\Rightarrow \boxed{B=-1}$

since $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} S_n = \frac{1}{3}$

Answer: $\boxed{\text{Converges to } 1/3}$

3. Determine if the series converges or diverges: $\sum_{k=1}^{\infty} ke^{-k^2}$ 10 pts.

$f'(x) = -2x^2e^{-x^2}+e^{-x^2}$
$= \underbrace{e^{-x^2}}_{>0}\underbrace{(-2x^2+1)}_{<0} < 0$

Integral Test $\left\{ f(x) = xe^{-x^2} = \frac{x}{e^{x^2}} \text{ is positive, continuous, and decreasing for } x\geq 1. \right.$

Recall: $\int e^u du = e^u + c$

$\int_1^{\infty} xe^{-x^2}dx = \lim_{b\to\infty}\int_1^b xe^{-x^2}dx = \lim_{b\to\infty}\int_{-1}^{-b^2} -\frac{1}{2}e^u du$

Let $u=-x^2$
$du=-2x\,dx$
$-\frac{1}{2}du = x\,dx$
$x=1 \Rightarrow u=-1$
$x=b \Rightarrow u=-b^2$

$= \lim_{b\to\infty}\left[-\frac{1}{2}e^u\right]_{-1}^{-b^2} = -\frac{1}{2}\lim_{b\to\infty}\left[e^{-b^2}-e^{-1}\right]$

$= -\frac{1}{2}\lim_{b\to\infty}\left[\frac{1}{e^{b^2}}-\frac{1}{e}\right] = \frac{1}{2e}$

Note: $e^{b^2}\to\infty$ as $b\to\infty$
so $\frac{1}{e^{b^2}}\to 0$ as $b\to\infty$

Answer: $\boxed{\text{Converges}}$ b/c the Integral Converges!