Quiz #3 score______/30 **Name** Key

Math 236

Spring 2008

SHOW WORK

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1. Evaluate: $\lim_{x\to 0} \dfrac{x-\ln(x+1)}{1-\cos 2x} \to \dfrac{0-0}{1-1} \to \dfrac{0}{0}$ Indet. form. → L'Hopital's Rule! 10 pts.

$\overset{H}{=} \lim_{x\to 0} \dfrac{1-\frac{1}{x+1}}{+2\sin 2x} \to \dfrac{1-1}{0} \to \dfrac{0}{0}$

$= \lim_{x\to 0} \dfrac{1-(x+1)^{-1}}{2\sin 2x} \to \dfrac{0}{0}$

$= \lim_{x\to 0} \dfrac{(x+1)^{-2}}{4\cos 2x} = \lim_{x\to 0} \dfrac{1}{4(x+1)^2\cos 2x} = \dfrac{1}{4\cdot 1\cdot 1} = \boxed{\dfrac{1}{4}}$

Answer ______

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2. Evaluate: $\lim_{x\to\infty} (1-3/x)^x \to (1-0)^\infty \to 1^\infty$ Indet. form. 10 pts.

Let $\boxed{y = \left(1-\frac{3}{x}\right)^x}$

So $\ln y = \ln\left(1-\frac{3}{x}\right)^x \Rightarrow \ln y = x\ln\left(1-\frac{3}{x}\right)$ so $\lim_{x\to\infty} \ln y = \lim_{x\to\infty} x\ln\left(1-\frac{3}{x}\right)$

So $\lim_{x\to\infty} x\ln\left(1-\frac{3}{x}\right) = \lim_{x\to\infty} \dfrac{\ln(1-3x^{-1})}{\frac{1}{x}} \to \dfrac{0}{0}$ $\to \infty\cdot 0$

$\overset{H}{=} \lim_{x\to\infty} \dfrac{\frac{1}{1-\frac{3}{x}}\cdot(3x^{-2})}{-x^{-2}} = \lim_{x\to\infty} \dfrac{-3}{1-\frac{3}{x}} = -3.$

0

So $\lim_{x\to\infty} \ln y = -3$ and ln a continuous fn $\Rightarrow \lim_{x\to\infty} \ln y = \ln\left(\lim_{x\to\infty} y\right)$ and so

$\ln\left(\lim_{x\to\infty} y\right) = -3 \Rightarrow \boxed{\lim_{x\to\infty} y = e^{-3}}$

Answer $\boxed{e^{-3}}$

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3. a) Determine if the sequence converges, and if so, what does it converge to? $\{\cos(3/n)\}_{n=1}^{\infty}$ 7 pts.

$\lim_{n\to\infty} \cos\frac{3}{n} = \cos 0 = \boxed{1}$ converges to

0

If $x = \frac{3}{n}$, $n\to\infty \Leftrightarrow x\to 0$

(since $\cos(x)$ is continuous $\lim_{x\to 0} \cos(x) = \cos\left(\lim_{x\to 0} x\right)$)

Answer $\boxed{1}$

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3. b) Now consider each of the following sequences and circle the correct answer for each in the boxes. 3 pts.

A. $\{\cos(n/3)\}_{n=1}^{\infty}$

as $n\to\infty$, $\cos\left(\frac{n}{3}\right)$ oscillates between -1 & 1 since $\frac{n}{3}\to\infty$

A. ***Converges or Diverges ?***

B. $\{(-1)^n[\cos(3/n)]\}_{n=1}^{\infty}$

$\frac{3}{n}\to 0$ as $n\to\infty$

$\lim_{n\to\infty} (-1)^n \cos\left(\frac{3}{n}\right)$; $(-1)^n \to \pm 1$, $\cos\left(\frac{3}{n}\right) \to 1$; bounces btwn -1 & 1

B. ***Converges or Diverges ?***

C. $\{(-1)^n[\sin(3/n)]\}_{n=1}^{\infty}$

$\frac{3}{n}\to 0$ as $n\to\infty$

$\lim_{n\to\infty} (-1)^n \sin\left(\frac{3}{n}\right) = 0$; $(-1)^n \to \pm 1$, $\sin\left(\frac{3}{n}\right) \to 0$

C. ***Converges or Diverges ?***