

1. A) Justify that the function $f(x) = 2x^5 + x^3 + 3x + 2$ has an inverse function on \mathbb{R} .

12 pts.

$$f'(x) = 10x^4 + 3x^2 + 3 > 0 \text{ on } \mathbb{R}$$

this is always $> 0 + 3$

so f is increasing on $\mathbb{R} \Rightarrow f$ is 1-1 on \mathbb{R}
 $\Rightarrow f^{-1}(x)$ exists.

B) Find $(f^{-1})'(2)$. Hint: Recall that we showed $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \forall x \in R_f$ provided the denominator is not zero.

$$\text{so } (f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(0)} = \frac{1}{3}$$

Note: $f(0) = 2$ i.e., $(0, 2)$ is on the graph of $y = f(x)$

$\Rightarrow f^{-1}(2) = 0$ i.e., $(2, 0)$ is on the graph of $y = f^{-1}(x)$.

ANSWER $(f^{-1})'(2) = \boxed{\frac{1}{3}}$

2. Rewrite $2\ln(x+1) + \frac{1}{3}\ln x - \ln(\cos(x))$ as a single logarithm.

6 pts.

$$= \ln(x+1)^2 + \ln x^{1/3} - \ln(\cos(x))$$

$$= \ln \left(\frac{(x+1)^2 x^{1/3}}{\cos(x)} \right)$$

ANSWER $\boxed{\ln \left(\frac{(x+1)^2 x^{1/3}}{\cos(x)} \right)}$

3. A) Find the domain of $f(x) = \ln(\ln x)$ and compute $f'(x)$.

$$f'(x) = \frac{1}{\ln x} \cdot \frac{1}{x}$$

12 pts.

$$D_f \Rightarrow \ln x > 0 \text{ and } x > 0$$

$$x > 1$$

domain of $f = \boxed{(1, \infty)}$

$$f'(x) = \boxed{\frac{1}{x \ln x}}$$

B) Find $f'(x)$ for the function $f(x) = e^{-5x^2}$.

$$f'(x) = e^{-5x^2} (-10x)$$

$$f'(x) = \boxed{-10x e^{-5x^2}}$$

C) Find the simplest exact value of $f(\ln(2))$ for the function $f(x) = e^x + 3e^{-x}$.

$$f(\ln(2)) = e^{\ln(2)} + 3e^{-\ln(2)}$$

$$= 2 + 3e^{\ln 2^{-1}}$$

$$= 2 + 3 \cdot 2^{-1} = 2 + 3 \cdot \frac{1}{2} = 2 + \frac{3}{2} = \frac{4}{2} + \frac{3}{2} = \frac{7}{2}$$

$$f(\ln(2)) = \boxed{\frac{7}{2}}$$