Strategies for integrating Products of Trigonometric Functions - I

Integrand	**m**	**n**	**Strategy: Use IBPs and reduction formulas with**
$\sin^m x \cos^n x$ $m \geq 0,\ n \geq 0$	m even	n even	$(\sin^2 x)^k = (1-\cos^2 x)^k$ *or* $(\cos^2 x)^k = (1-\sin^2 x)^k$ *or* use $\sin x \cos x = \frac{1}{2}\sin 2x$ *or* $u-$substitution *or* $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$ *or* $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$ *or IBPs*
	odd m ≥ 3	n anything	$\sin^m x = \sin^{m-1} x \sin x$ $\sin^{m-1} x = \sin^{2k} x$ $= (\sin^2 x)^k = (1-\cos^2 x)^k$ Now expand $(1-\cos^2 x)^k$.
	m anything	odd n ≥ 3	$\cos^n x = \cos^{n-1} x \cos x$ $\cos^{n-1} x = \cos^{2k} x$ $= (\cos^2 x)^k = (1-\sin^2 x)^k$ Now expand $(1-\sin^2 x)^k$.
$\sin mx \cos nx$ $\sin mx \sin nx$ $\cos mx \cos nx$ $m \neq n$			$\sin A\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$ $\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ $\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

In all cases above and below (unless otherwise noted), use u-substitutions to evaluate the resulting integrals!!

Strategies for integrating Products of Trigonometric Functions –II

Integrand	**m**	**n**	**Strategy: Use**
$\tan^m x \sec^n x$ $m > 0,\ n > 0$	m anything	n even	$\sec^n x = \sec^{n-2} x \sec^2 x$ $\sec^{n-2} x = \sec^{2k} x$ $= (\sec^2 x)^k = (1+\tan^2 x)^k$ Now expand $(1+\tan^2 x)^k$
	m odd	n anything	$\tan^m x \sec^n x = \tan^{m-1} x \sec^{n-1} x \sec x \tan x$ $\tan^{m-1} x = \tan^{2k} x$ $= (\tan^2 x)^k = (\sec^2 x - 1)^k$ Now expand $(\sec^2 x - 1)^k$.
	m even	n odd	$\tan^m x = \tan^{2k} x$ $= (\tan^2 x)^k = (\sec^2 x - 1)^k$ Now expand $(\sec^2 x - 1)^k$, and use Reduction Formulas or IBPs.
$\cot^m x \csc^n x$ $m > 0, n > 0$			Same ideas as above for $\tan^m x \sec^n x$, except use the following trig. identities: $\csc^2 x = \cot^2 x + 1$ $\cot^2 x = \csc^2 x - 1$

Reduction Formulas:

$$\int \sin^n(x)dx = -\tfrac{1}{n}\sin^{n-1}(x)\cos(x) + \tfrac{n-1}{n}\int \sin^{n-2}(x)dx$$

$$\int \cos^n(x)dx = \tfrac{1}{n}\cos^{n-1}(x)\sin(x) + \tfrac{n-1}{n}\int \cos^{n-2}(x)dx$$

$$\int \tan^n(x)dx = \tfrac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)dx$$

$$\int \sec^n(x)dx = \tfrac{1}{n-1}\sec^{n-2}(x)\tan(x) + \tfrac{n-2}{n-1}\int \sec^{n-2}(x)dx$$

$$\int \csc^n(x)dx = -\tfrac{1}{n-1}\csc^{n-2}(x)\cot(x) + \tfrac{n-2}{n-1}\int \csc^{n-2}(x)dx$$

$$\int \cot^n(x)dx = -\tfrac{1}{n-1}\cot^{n-1}(x) - \int \cot^{n-2}(x)dx$$