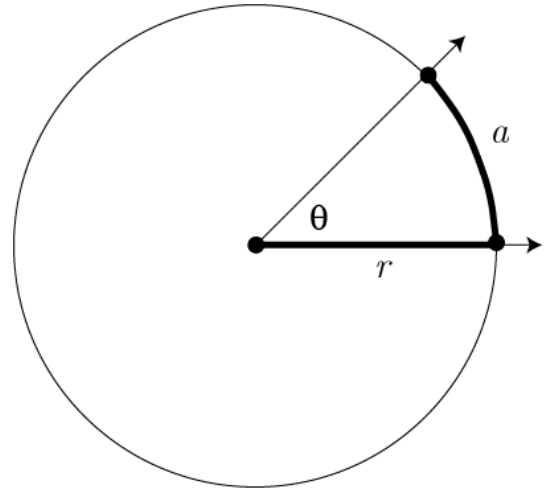


# Special Inequalities for Sine Worksheet

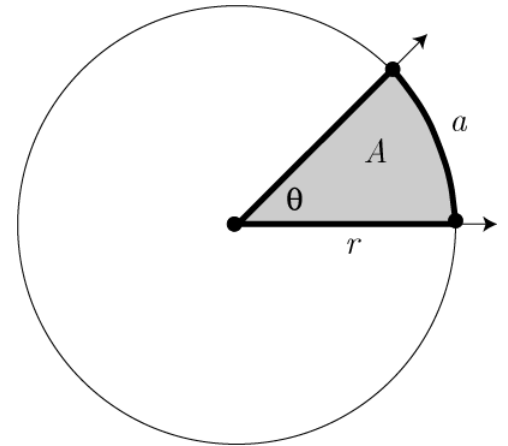
First recall given two radii of a circle of radius  $r$  that the **radian** measure of the angle  $\theta$  "subtended" by the radii is defined to be the ratio of the length of the enclosed arc  $a$  to the radius  $r$  of the circle; that is,

$$\theta = \frac{a}{r}.$$

1. Then  $a =$  \_\_\_\_\_
2. The **Circumference** of the circle is \_\_\_\_\_
3. The **Area** of the circle is \_\_\_\_\_



Recall also from geometry that the ratio of the *area of the sector*  $A$  to the *area of the circle* is equal to the ratio of the length of the arc  $a$  to the *circumference of the circle*. Use this to determine a formula that represents  $A$  in terms of  $r$  and  $\theta$ .



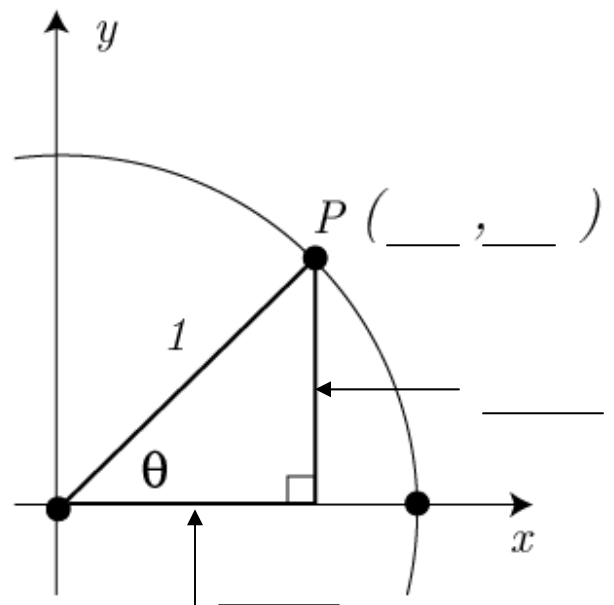
4.  $A =$   
\_\_\_\_\_

5. Use what you learned in trigonometry to label the coordinates of the point  $P$  in the unit circle shown in the figure to the right.

6. Again use what you learned in trigonometry to properly mark the lengths of the right triangle in the figure to the right.

7. Find an expression in terms of  $\theta$  for the *slope* of the line running through the points  $(0,0)$  and  $P$ .

slope =  
\_\_\_\_\_



8. Find the *equation* of the line running through the points  $(0,0)$  and  $P$ .

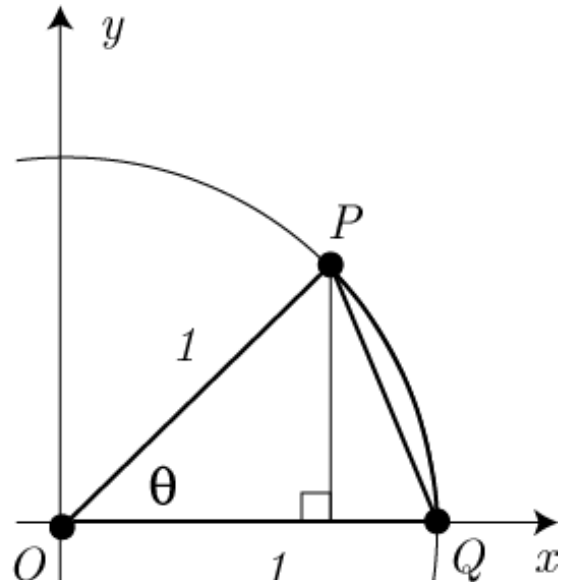
$y =$   
\_\_\_\_\_

9. Use what you learned in geometry to find an expression in terms of  $\theta$  for the *area of the triangle POQ* in the unit circle shown in the figure to the right.

Area triangle POQ = \_\_\_\_\_

10. Use what you found in question 4 to find an expression in terms of  $\theta$  for the *area of the sector POQ* in the unit circle shown in the figure to the right.

Area sector POQ = \_\_\_\_\_



11. Which has more area, *triangle POQ* or *sector POQ*? \_\_\_\_\_

12. Using the answers in questions 9-11 and the fact that both areas are positive we can establish the following inequality,

$0 < \text{_____} < \text{_____}$
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13. Use the answer to question 8 to help find coordinates of the point *D* shown in the figure to the right.

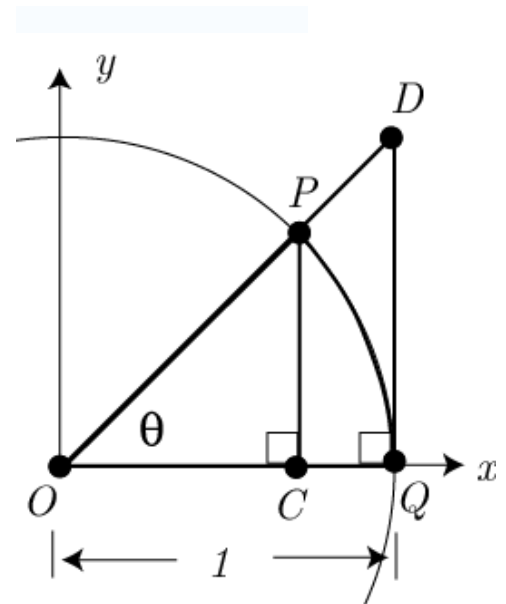
$$D = (\text{_____, } \text{_____})$$

14. Use what you learned in geometry to find an expression in terms of  $\theta$  for the *area of the right triangle DOQ* in the unit circle shown in the figure to the right.

Area triangle DOQ = \_\_\_\_\_

15. Use what you learned in geometry to find an expression in terms of  $\theta$  for the *area of the right triangle POC* in the unit circle shown in the figure to the right.

Area triangle POC = \_\_\_\_\_



16. Arrange the following from least to greatest, the *Area triangle DOQ*, *Area triangle POC*, and *Area sector POQ*, and establish the inequality using the answers from questions 10, 14 and 15:

$\text{_____} \leq \text{_____} \leq \text{_____}$
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