Quiz #4 score____/30 Name Key

Calculus I

Fall 2009

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1. **A) Use your favorite limit definition of derivative to find $\frac{df}{dx}$ if $f(x)=\frac{1}{\sqrt{x+1}}$. Do not skip steps!:** 17 pts.

$$f'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x}$$

$$f'(x)=\lim_{z\to x}\frac{\frac{1}{\sqrt{z+1}}-\frac{1}{\sqrt{x+1}}}{z-x}$$

$$=\lim_{z\to x}\frac{\sqrt{x+1}-\sqrt{z+1}}{\sqrt{z+1}\cdot\sqrt{x+1}}\cdot\frac{1}{z-x}$$

$$=\lim_{z\to x}\frac{\sqrt{x+1}-\sqrt{z+1}}{\sqrt{z+1}\cdot\sqrt{x+1}}\cdot\frac{\sqrt{x+1}+\sqrt{z+1}}{\sqrt{x+1}+\sqrt{z+1}}\cdot\frac{1}{z-x}$$

$$=\lim_{z\to x}\frac{(x+1)-(z+1)}{(\sqrt{z+1}\sqrt{x+1})(\sqrt{x+1}+\sqrt{z+1})}\cdot\frac{1}{z-x}$$

$$=\lim_{z\to x}\frac{x+1-z-1}{(\sqrt{z+1}\sqrt{x+1})(\sqrt{x+1}+\sqrt{z+1})}\cdot\frac{1}{z-x}$$

$$=\lim_{z\to x}\frac{-\cancel{(z-x)}}{(\sqrt{z+1}\sqrt{x+1})(\sqrt{x+1}+\sqrt{z+1})}\cdot\frac{1}{\cancel{z-x}}$$

$$=\lim_{z\to x}\frac{-1}{(\sqrt{z+1}\sqrt{x+1})(\sqrt{x+1}+\sqrt{z+1})}$$

$$=\frac{-1}{(\sqrt{x+1}\sqrt{x+1})(\sqrt{x+1}+\sqrt{x+1})}$$

$$=\frac{-1}{(x+1)\sqrt{x+1}+\sqrt{x+1}}$$

$$\left(\frac{-1}{2(x+1)\sqrt{x+1}}\right)$$

$$//$$

$$=\boxed{\frac{-1}{(x+1)(2\sqrt{x+1})}=f'(x)}$$

$$f'(0)=\frac{-1}{(0+1)2\sqrt{0+1}}=\frac{-1}{2\sqrt{1}}=-\frac{1}{2}$$

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1. **B) Now, what is $f'(0)=$ __$-1/2$__ ?** 3 pts.

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1. **C) Now give the equation of the tangent line to the graph of f at $(c, f(c))$ for $c=0$.** 6 pts.

Give the equation in point-slope form!!

$$f(0)=\frac{1}{\sqrt{0+1}}=\frac{1}{\sqrt{1}}=1$$

$$(y-y_1)=m(x-x_1)$$

$$\boxed{(y-1)=-\tfrac{1}{2}(x-0)}$$

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1. **D) For what values of x is f continuous?** (*Hint*: Remember that we showed that the square root function is continuous on its domain; so what does that mean about where f is continuous?) 4 pts.

Then, f is continuous everywhere it is defined. So for $f(x)=\frac{1}{\sqrt{x+1}}$, must have $x+1>0 \Rightarrow \boxed{x>-1}$ OR $\boxed{(-1,\infty)}$.