

1. Give the equivalent δ, ϵ statement for $\lim_{x \rightarrow \frac{1}{3}} \frac{1}{x} = 3$.

10 pts.

$$\forall \epsilon > 0 \exists \delta > 0 \text{ s.t. } 0 < |x - \frac{1}{3}| < \delta \Rightarrow |\frac{1}{x} - 3| < \epsilon$$

2. Now Prove: $\lim_{x \rightarrow \frac{1}{3}} \frac{1}{x} = 3$ (with a δ, ϵ proof of course, supplying reasons for each step!)

20 pts.

Proof: let $\epsilon > 0$ choose $\delta = \min\left\{\frac{1}{4}, \frac{\epsilon}{36}\right\}$

let $0 < |x - \frac{1}{3}| < \delta$ by hypothesis

so $x \neq \frac{1}{3}$ and $|x - \frac{1}{3}| < \delta$ by alg

so $|x - \frac{1}{3}| < \frac{\epsilon}{36}$ by def of min

so $36|x - \frac{1}{3}| < \epsilon$ by alg*

since $|x - \frac{1}{3}| < \frac{1}{4}$ also by def of min

then $-\frac{1}{4} < x - \frac{1}{3} < \frac{1}{4}$ by alg

so $0 < \frac{1}{12} < x < \frac{7}{12}$ by alg

so $\frac{12}{7} < \frac{1}{x} < 12$ by Reciprocal law of inequality

so $\frac{1}{|x|} < 12$ by alg and $\frac{3}{|x|} < 36$ by alg

so $|x - \frac{1}{3}| \cdot \frac{3}{|x|} < 36|x - \frac{1}{3}|$ by alg

so $|x - \frac{1}{3}| \cdot \frac{3}{|x|} < 36|x - \frac{1}{3}| < \epsilon$ (Refer to *)

so $\frac{3|x - \frac{1}{3}|}{|x|} < \epsilon$ by transitive law of inequality

so $|-3||x - \frac{1}{3}| < \epsilon$ by alg

so $|-3(x - \frac{1}{3})| < \epsilon$

so $|1 - 3x| < \epsilon$

so $|\frac{1-3x}{x}| < \epsilon$

so $|\frac{1}{x} - 3| < \epsilon$

Oh Yeah!!

scrap:

$$|\frac{1}{x} - 3| < \epsilon$$

$$\Leftrightarrow |\frac{1}{x} - \frac{3x}{x}| < \epsilon$$

$$\Leftrightarrow |\frac{1-3x}{x}| < \epsilon$$

$$\Leftrightarrow \frac{|1-3x|}{|x|} < \epsilon$$

$$\Leftrightarrow \frac{|-3(x - \frac{1}{3})|}{|x|} < \epsilon$$

$$\Leftrightarrow \frac{|-3||x - \frac{1}{3}||}{|x|} < \epsilon$$

$$\Leftrightarrow \frac{3|x - \frac{1}{3}|}{|x|} < \epsilon$$

$$\Leftrightarrow \frac{3}{|x|} \cdot |x - \frac{1}{3}| < \epsilon$$

Assume $\frac{1}{4}$ Gold $|x - \frac{1}{3}| < \frac{1}{4} \Leftrightarrow -\frac{1}{4} < x - \frac{1}{3} < \frac{1}{4}$

$$\Leftrightarrow 0 < \frac{1}{12} < x < \frac{7}{12}$$

$$\Leftrightarrow \frac{12}{7} < \frac{1}{x} < 12$$

$$\Leftrightarrow \frac{1}{|x|} < 12 \Leftrightarrow \frac{3}{|x|} < 36$$

$$\Leftrightarrow \frac{3|x - \frac{1}{3}|}{|x|} < 36|x - \frac{1}{3}| < \epsilon$$

$$\Leftrightarrow 36|x - \frac{1}{3}| < \epsilon$$

$$\Leftrightarrow |x - \frac{1}{3}| < \frac{\epsilon}{36}$$