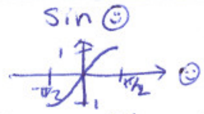
Quiz #5 score______/35 **Name** Key
Math 232
Fall 2008

SHOW WORK where appropriate! NO CALCULATORS!!

---

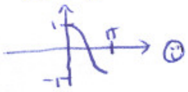
1. Complete each of the following. 7 pts.



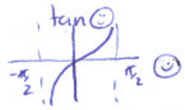
y is the angle here!

a) $y=\sin^{-1}x$ if and only if $\sin y =$ $\underline{\;x\;}$ $\forall\, x\in$ $\underline{[-1,1]}$ *and* $y\in$ $\underline{[-\pi/2,\pi/2]}$.

cos ☺



b) $\cos^{-1}(\cos x)=$ $\underline{\;x\;}$ $\forall x\in$ $\underline{[0,\pi]}$.



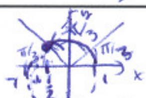
c) $\underline{\tan^{-1}x}$ is the angle in the interval $\underline{(-\pi/2,\pi/2)}$ whose tangent is x.

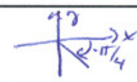
$\theta=\tan^{-1}(x)$
$\tan\theta = x$

---

2. Calculate each of the following. 6 pts.

$\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

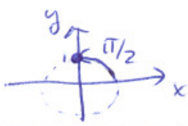




a) $\sin^{-1}(1)=$ $\underline{\pi/2}$

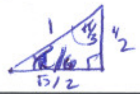
$\theta=\sin^{-1}(1)$, $\theta\in[-\pi/2,\pi/2]$
$\sin\theta = 1$

$\boxed{\theta=\pi/2}$



b) $\cos^{-1}(-\frac{1}{2})=$ $\underline{2\pi/3}$

$\theta=\cos^{-1}(-\frac{1}{2})$, $\theta\in[0,\pi]$ quadrants I, II
$\cos\theta=-\frac{1}{2}$ quadrant II since <0



$\cos\pi/3 = 1/2$

c) $\tan^{-1}(-1)=$ $\underline{-\pi/4}$

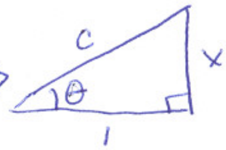
$\theta=\tan^{-1}(-1)$, $\theta\in(-\pi/2,\pi/2)$ quadrants I + IV
$\tan\theta=-1$
so $\frac{\sin\theta}{\cos\theta}=-1$ ← b/c negative, the angle θ must be in quadrant IV

---

3. Simplify the following to an algebraic expression: 4 pts.

$\sec^2(\tan^{-1}x)=$ $\underline{1+x^2}$

$\theta=\tan^{-1}x$
$\tan\theta = x = \frac{x}{1} = \frac{opp}{adj} \Rightarrow$



$\boxed{1^2+x^2=c^2}$

so $\sec\theta = \frac{hyp}{adj} = \frac{c}{1} = c$.

$\sec^2(\theta)=(\sec\theta)^2 = c^2$

---

4. Calculate the derivatives of the following functions. 12 pts.

a) $f(x)=\sin^{-1}(3x^2)$

$$f'(x)=\frac{1}{\sqrt{1-(3x^2)^2}}\cdot 6x = \frac{6x}{\sqrt{1-9x^4}}$$

b) $f(x)=\sec^{-1}(e^x)$

$$f'(x)=\frac{1}{|e^x|\sqrt{(e^x)^2-1}}\cdot e^x = \frac{e^x}{e^x\sqrt{e^{2x}-1}} = \frac{1}{\sqrt{e^{2x}-1}}$$

e^x is always positive!

c) $f(x)=(\cos(2x))^{-1}$

$$f'(x) = -(\cos(2x))^{-2}\cdot(-\sin(2x))\cdot 2 = \frac{2\sin(2x)}{\cos^2(2x)}$$

d) $f(x)=\sec(1+\tan^{-1}x)$

$$f'(x)=\sec(1+\tan^{-1}x)\tan(1+\tan^{-1}x)\cdot\left(\frac{1}{1+x^2}\right)$$

---

5) Prove that the derivative of the inverse cosine function is $\frac{-1}{\sqrt{1-x^2}}$. 6 pts.

$$\cos(\cos^{-1}(x)) = x$$

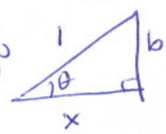
$$\frac{d}{dx}\left(\cos(\cos^{-1}(x))\right) = \frac{d}{dx}(x)$$

$$-\sin(\cos^{-1}(x))\cdot(\cos^{-1}(x))' = 1$$

so
$$(\cos^{-1}(x))' = \frac{1}{-\sin(\cos^{-1}(x))} = \frac{1}{-\sqrt{1-x^2}} = \frac{-1}{\sqrt{1-x^2}}.$$

simplify $\sin(\cos^{-1}(x))$ = $\sin\theta$

But $\theta=\cos^{-1}x$, $\theta\in[0,\pi]$
$\Rightarrow \cos\theta = x = \frac{x}{1} = \frac{adj}{hyp}$



$\sin\theta = \frac{b}{1} > 0$
so $\sin\theta = \sqrt{1-x^2}$
so $\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$

$x^2+b^2=1$
$b^2=1-x^2$
$b=\pm\sqrt{1-x^2}$