Quiz #4 score_____/35 **Name** Key
Math 232
Fall 2008

SHOW WORK where appropriate! NO CALCULATORS!!

1. Circle all of the following forms that are *indeterminant*. For the rest, indicate what it goes to. 6 pts.

$\infty^{\infty} \to \infty$ $\quad \infty^{-\infty} \to \frac{1}{\infty^{\infty}} \to \frac{1}{\infty} \to 0$ $\quad 0^{\infty} \to 0$ $\quad$ (circled) ∞^0

(circled) 1^{∞} $\quad 0^{-\infty} \to \frac{1}{0^{\infty}} \to \frac{1}{0} \to \infty$ $\quad$ (circled) 0^0 $\quad \infty^1 \to \infty$

$1^0 \to 1$ $\quad$ (circled) $1^{-\infty} \to \frac{1}{1^{\infty}}$ indet. $\quad 0^1 \to 0$ $\quad 0^{-1} \to \frac{1}{0} \to \infty$

2. Complete each of the following. 6 pts.

a) If $\lim_{x\to 3} \ln(f(x)) = 0$, then $\lim_{x\to 3} f(x) =$ 1.

b) If $\lim_{x\to 0} \ln(f(x)) \to -\infty$, then $\lim_{x\to 0} f(x) =$ 0.

c) $\sin(60^\circ) =$ $\frac{\sqrt{3}}{2}$ $\quad = \frac{\text{opp}}{\text{hyp}}$

d) $\sec(45^\circ) =$ $\sqrt{2}$ $\quad = \frac{1}{\cos(45^\circ)}$ and $\cos(45^\circ) = \frac{1}{\sqrt{2}}$

e) $\tan(30^\circ) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

f) **True or False:** $\frac{\csc\theta}{\sec\theta} = \cot\theta$ $\quad \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$

3. Calculate each limit: 8 pts.

a) $\lim_{x\to\infty} x^{\frac{1}{x}} \to \infty^0$ so, indeterminant $\lim_{x\to\infty} \ln x^{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{1}{x}\ln x = \lim_{x\to\infty} \frac{\ln x}{x} \to \frac{\infty}{\infty}$

L'H $= \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0.$

So $\lim_{x\to\infty} x^{\frac{1}{x}} = e^0 = 1$

b) $\lim_{x\to 0^+} x^{\ln x} \to 0^{-\infty} \to \frac{1}{0^{\infty}} \to \frac{1}{0} \to \infty$

Not indeterminant!

4) For each of the following problems, determine the *exact* solution. 7 pts.

a) Solve $2^x = 3^{x+1}$.

$\ln 2^x = \ln 3^{x+1}$

$x \ln 2 = (x+1)\ln 3$

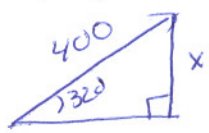
so $x\ln 2 = x\ln 3 + \ln 3$

so $x\ln 2 - x\ln 3 = \ln 3$

so $x(\ln 2 - \ln 3) = \ln 3 \Rightarrow$ $x = \frac{\ln 3}{\ln 2 - \ln 3}$

OR $x = \frac{\ln 3}{\ln(2/3)}$

b) Alina is flying her kite and has let out 400 feet of kite string. If the angle made by the ground and the line of the kite string is 32°, how high is the kite?



$\sin(32^\circ) = \frac{x}{400}$

So $x = 400 \cdot \sin(32^\circ)$ ft.

5) For each of the following functions, determine $f'(x)$. You do not need to simplify your answers! 8 pts.

a) $f(x) = x^{\ln x}$ ① ② Log. Diff!!

① $\ln f(x) = \ln(x^{\ln x})$

so $\ln f(x) = (\ln x)\cdot\ln x$

$= (\ln x)^2$

② so $\frac{1}{f(x)} \cdot f'(x) = 2(\ln x)\cdot\frac{1}{x}$

so $f'(x) = \frac{2}{x}(\ln x)\cdot x^{\ln x}$

b) $f(x) = \frac{e^{2x}(x^3-2)^4}{x(3e^{5x}+1)}$ ① $\ln f(x) = \ln\left(e^{2x}(x^3-2)^4\right) - \ln\left(x(3e^{5x}+1)\right)$

so

$\ln f(x) = \ln e^{2x} + \ln(x^3-2)^4 - (\ln x + \ln(3e^{5x}+1))$

so

$\ln f(x) = 2x + 4\ln(x^3-2) - \ln x - \ln(3e^{5x}+1)$

② $\frac{1}{f(x)} f'(x) = 2 + \frac{4(3x^2)}{(x^3-2)} - \frac{1}{x} - \frac{1\cdot(3e^{5x}\cdot 5)}{3e^{5x}+1}$

so $f'(x) = \left(2 + \frac{12x^2}{x^3-2} - \frac{1}{x} - \frac{15e^{5x}}{3e^{5x}+1}\right) f(x)$