

SHOW WORK where appropriate! NO CALCULATORS!!

1. Circle all of the following forms that are *indeterminant*. For the rest, indicate what it goes to. 6 pts.

$\infty + \infty \rightarrow \infty$

$\infty - \infty$ (circled)

$0 \cdot \infty$ (circled)

$\infty \cdot \infty \rightarrow \infty$

$\frac{\infty}{\infty}$ (circled)

$\frac{\infty}{0} \rightarrow \infty$

$\frac{0}{\infty} \rightarrow 0$

$\frac{0}{0}$ (circled)

$\frac{\infty}{1} \rightarrow \infty$

$\frac{1}{0} \rightarrow \infty$

$\frac{0}{1} \rightarrow 0$

$\frac{1}{\infty} \rightarrow 0$

2. Complete each of the following. Simplify as much as possible: 6 pts.

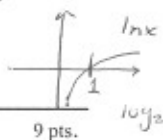
a) $\log_b 1 = 0$

b) $\ln e = 1$

c) $\ln\left(\frac{1}{e^2}\right) = \ln(e^{-2}) = -2 \ln e = -2$

d) $\log_2 6 - \log_2 3 = \log_2\left(\frac{6}{3}\right) = \log_2(2) = 1$

e) The domain of $f(x) = \sqrt{\ln(x-2)}$ is $[3, \infty)$
 $x-2 > 0 \Rightarrow x > 2$
 $\ln(x-2) \geq 0 \Rightarrow x-2 \geq 1$
 $\Rightarrow x \geq 3$



3. Calculate each limit: 9 pts.

a) $\lim_{x \rightarrow \infty} \frac{e^{3x}}{1-e^{2x}} \rightarrow \frac{\infty}{-\infty}$
 L'H $= \lim_{x \rightarrow \infty} \frac{3e^{3x}}{-2e^{2x}}$
 $= \lim_{x \rightarrow \infty} -\frac{3}{2}e^x \rightarrow -\infty$

b) $\lim_{x \rightarrow 3} \frac{2^x - 8}{3 - x} \rightarrow \frac{0}{0}$
 L'H $= \lim_{x \rightarrow 3} \frac{(\ln 2)2^x}{-1}$
 $= -(\ln 2) \cdot 2^3 = -8 \ln 2$

c) $\lim_{x \rightarrow 0^+} \frac{x}{\log_2 x} \rightarrow \frac{0^+}{-\infty}$
 $= 0$

4) Solve the equation exactly: $\log_2\left(\frac{x-1}{x+1}\right) = 4$ 5 pts.

$\log_2\left(\frac{x-1}{x+1}\right) = 4$
 $\Rightarrow \frac{x-1}{x+1} = 2^4 = 16$
 $\Rightarrow x-1 = 16(x+1)$
 $\Rightarrow x-1 = 16x+16$
 $\Rightarrow -17 = 15x$
 $\Rightarrow x = -\frac{17}{15}$

5) For each of the following functions, compute $f'(x)$. Simplify if possible. 9 pts.

a) $f(x) = \sqrt{\ln x} = (\ln x)^{1/2}$
 $f'(x) = \frac{1}{2} (\ln x)^{-1/2} \cdot \frac{1}{x}$
 $= \frac{1}{2x \sqrt{\ln x}}$

b) $f(x) = \log_2(3^x) \stackrel{g(x)=3^x}{\Rightarrow}$
 $f'(x) = \frac{1}{(\ln 2) 3^x} \cdot (\ln 3) 3^x$
 $= \frac{\ln 3}{\ln 2} = \log_2 3$
 or: $f(x) = x \log_2(3) = (\log_2 3) x$
 so $f'(x) = \log_2(3)$

c) $f(x) = x^2 \ln(\ln x)$
 $f'(x) = x^2 \cdot [\ln(\ln x)]' + 2x \ln(\ln x)$
 $= x^2 \cdot \frac{1}{\ln x} (\ln x)' + 2x \ln(\ln x)$
 $= x^2 \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + 2x \ln(\ln x)$
 $= \frac{x}{\ln x} + 2x \ln(\ln x) = x \left[\frac{1}{\ln x} + 2 \ln(\ln x) \right]$