

SHOW WORK and JUSTIFY your answer to all parts using appropriate calculus! NO CALCULATORS!!

1. Determine if the following limit exists. If it doesn't exist, say where the limit tends if possible.

5 pts.

$$\lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x-1}} \rightarrow \frac{1}{\sqrt{0}} \rightarrow \frac{1}{0^+} \rightarrow +\infty$$

\uparrow
x is getting close to one but is bigger than 1.
D.N.E.

Answer: $\lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x-1}} \rightarrow +\infty$

2. Given that f is a differentiable function, determine a formula for the derivative of $\frac{1}{f(x)}$. or $[f(x)]^{-1}$, so 5 pts.

Quotient Rule!

$$\left(\frac{1}{f(x)}\right)' = \frac{f(x) \cdot (1)' - (1) f'(x)}{[f(x)]^2}$$

$$= \frac{f(x) \cdot 0 - f'(x)}{[f(x)]^2} = \frac{0 - f'(x)}{[f(x)]^2}$$

by chain-rule: $-1[f(x)]^{-2} f'(x)$

$$\left(\frac{1}{f(x)}\right)' = \frac{-f'(x)}{[f(x)]^2}$$

3. Differentiate each of the following functions using the appropriate rules. DO NOT SIMPLIFY YOUR ANSWERS.

10 pts.

A) $f(x) = \sqrt{3-4x^6} = (3-4x^6)^{1/2}$

$$f'(x) = \frac{1}{2} (3-4x^6)^{-1/2} (3-4x^6)'$$

so $f'(x) = \frac{1}{2} (3-4x^6)^{-1/2} (-24x^5)$ $f'(x) = -12 (3-4x^6)^{-1/2} x^5$

B) $f(x) = x(3x^2+1)^9$

$$f'(x) = x [9(3x^2+1)^8 (3x^2+1)'] + 1 \cdot (3x^2+1)^9$$

so $f'(x) = x [9(3x^2+1)^8 (6x)] + (3x^2+1)^9$ $f'(x) = 54x^2 (3x^2+1)^8 + (3x^2+1)^9$

4) Given that f and g are differentiable functions, give a formula for the derivative of $x^2 f(g(x))$.

10 pts.

$$x^2 [f(g(x))] + (x^2)' f(g(x))$$

$$x^2 f'(g(x)) g'(x) + 2x f(g(x))$$

Answer: $\frac{d}{dx}(x^2 f(g(x))) = x^2 f'(g(x)) g'(x) + 2x f(g(x))$

5) Given the equation $xy^2 + 3x^2 = 4$, use implicit differentiation to find $\frac{dy}{dx}$.

5 pts.

$$\frac{d}{dx}(xy^2 + 3x^2) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}(xy^2) + \frac{d}{dx}(3x^2) = 0$$

$$x \cdot (2yy') + 1 \cdot y^2 + 6x = 0$$

$$2xyy' = -6x - y^2$$

$$\text{so } y' = \frac{-6x - y^2}{2xy}$$

Answer: $\frac{dy}{dx} = \frac{-6x - y^2}{2xy}$