Test #3
Math 232
FALL 2008

MUST SHOW WORK FOR EACH PROBLEM!

score____/150 **Name**____

Extra Credit: Name Bart Simpson's sisters.

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1. Let f be continuous on $[a,b]$.

(A) Then, provided the following limit exists, the **definite integral** of f from a to b is defined in terms of a Riemann sum to be: **16 pts**

(Completely define everything you use!)

$$\int_a^b f(x)\,dx \equiv$$

(B) Complete: $h(x)$ is an **antiderivative** of a function $g(x)$ if the following equation holds: ______________________________

(C) **The Fundamental Theorem of Calculus** quite remarkably establishes the following result: ***(Be sure to completely define any functions or notations you use!)***

$$\int_a^b f(x)\,dx =$$

2. Evaluate the integrals. **24 pts**

A) $\int e^{5x}dx$

B) $\int_0^{\pi} \sec x \tan x\,dx$

C) $\int 3x\cos(x^2+12)\,dx$

D) $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx$

E) $\int_0^1 15(x^2-2)^2\,dx$

F) $\int_1^e \frac{\ln x}{x}\,dx$

3. Find the derivative of the following functions. DO NOT SIMPLIFY: **10 pts**

A) $f(x) = \sec^{-1}\left((\tan x)^{-1}\right)$

B) $f(x) = \int_{x^2}^{4} \sqrt{1+t^2}\,\,dt$

4. **Circle** all concepts below that were important in our proof in class of the given result for each of the following: **10 pts**

(A) *The Fundamental Theorem of Calculus (FTC-I).*

Mean Value Theorem
Mean Value Theorem for Integrals
Limit Definition of the Derivative
Limit Definition of the Definite Integral
Algebraic properties of definite integrals
Limit of a constant is a constant
Subdividing an interval into equally spaced subintervals
Continuity of the integrand function.

(B) *The 2nd Fundamental Theorem of Calculus (FTC-II).*

Mean Value Theorem
Mean Value Theorem for Integrals
Limit Definition of the Derivative
Limit Definition of the Definite Integral
Algebraic properties of definite integrals
Limit of a constant is a constant
Subdividing an interval into equally spaced subintervals
Continuity of the integrand function.

5. **Derive** the derivative formula for the inverse cotangent function. *Hint*: Start with $\cot\left(\cot^{-1}(x)\right) =$ ________ **7 pts**

6. Evaluate the following integrals: **16 pts**

A) $\int \frac{3-x}{x^3}\,dx$

B) $\int \cot x\,dx$

C) $\int \frac{1}{x\left(1+[\ln(x)]^2\right)}\,dx$

D) $\int \frac{\sin(1+e^{-x})}{e^x}\,dx$

7. Answer the following: **17 pts**

A) $\cos\left(\cos^{-1}(x)\right) =$ ________ for all $x \in$ ____________

B) $\sin^{-1}\left(\sin(x)\right) =$ ________ for all $x \in$ ____________

C) $\cos^{-1}(0) = ?$ **circle one of :** $0 \quad 1 \quad -1 \quad \frac{\pi}{2} \quad \pi$ DNE

D) $\cos^{-1}(\pi) = ?$ **circle one of :** $0 \quad 1 \quad -1 \quad \frac{\pi}{2} \quad \pi$ DNE

E) $\cos^{-1}\left(\cos(3\pi)\right) =$ ______

F) $\tan\left(\cos^{-1}(x)\right) =$ ______________

G) $y = \tan^{-1} x$ iff $\tan y =$ ____

$\forall\ x \in$ _______ *and* $y \in$ _______.

H) $\cos\left(\tan^{-1}(x)\right) =$ ______________

8. *Set up, but **do not evaluate*** each of the following integrals: **15 pts**

(A) The *signed area* between the graph of $y = \sin x$ and the x-axis on the interval $[-\pi, \frac{3\pi}{2}]$.

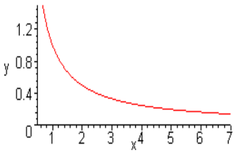
(B) The *true area* between the graph of $y = \sin x$ and the x-axis on the interval $[-\pi, \frac{3\pi}{2}]$.

(C) The *area* between the graphs of $y = \sin x$ and $y = \cos x$ on the interval $[0, \frac{\pi}{2}]$.

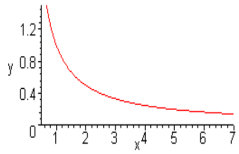
(D) The *area* between the graph of $y = \sin^2 x$ and $y = 0$ on the interval $[0, x]$ where $x > 0$.

9. ***Sketch*** each of the following approximations to $\int_1^7 \frac{1}{x}\,dx$ with $n = 3$ subdivisions and then ***compute*** *(calculators OK for A)—D)!).* **24 pts**

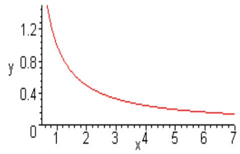
A) *Left-Hand Sum*



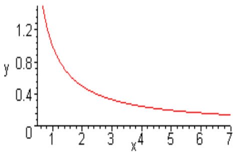
B) *Lower Sum*



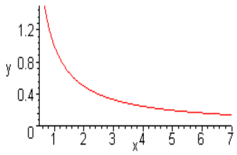
C) *Trapezoid Rule*



D) *Midpoint Rule*



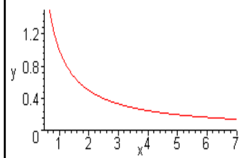
E) ***Sketch** and **compute** the exact area:* $\int_1^7 \frac{1}{x}\,dx$, then give a calculator approximation to the value.



area =

area ≈

F) *Determine the <u>value of c</u> from the MVTI* for $f(x) = \frac{1}{x}$ on $[1,7]$. *Sketch the rectangle* with height $f(c)$ whose area is exactly that in E).



c =

c ≈

10. True/False. Circle one. **6 pts**

A) $\int \sec x \; dx = -\ln|\sec x + \tan x| + C$ ....**True** or **False**

B) If $G'(x) = f(x)$ for all $x \in \mathbb{R}$ then $\int f(x)\; dx = G(x)\Big|_a^b$ ....**True** or **False**

C) The **average value** of the function $f(x) = \sec^2(x)$ on the interval $[0, \pi/4]$ is $\frac{4}{\pi}$ ....**True** or **False**

11. **Prove** that if f and g are continuous on $[a,b]$, then $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$. **5 pts**