Test #2
Math 232
Fall 08

MUST SHOW WORK FOR EACH PROBLEM!

score____/150

Name________________________________

Extra Credit: *Name the TV show and character famous for this regular exclamation:* "Look boss!! De plane! De plane!"________________________

15 pts 1. A) Determine the domain of $f(x)=\frac{x}{\ln x-1}$.

ANS________________

1. B) Determine any points on the graph of $f(x)=\frac{x}{\ln x-1}$ where the graph has a horizontal tangent line.

ANS_________________

30 pts 2. **Quick Proofs:**

A) ***Prove*** that $\frac{d}{dx}(\ln x)=\frac{1}{x}$ by completing the following equation and then differentiating it: $e^{\ln x}=$ ____.

B) Use the limit definition of derivative to **derive** the derivative of $f(x)=\cos x$. *Give a brief reason after each step.* I'll start you off:

$f'(x)\equiv\lim_{h\to 0}$ *by definition of derivative*

C) **Prove** the result that $(\sec x)'=\sec x\tan x$. Use may use the result of part B) above.

D) **Prove** the trig identity: $\csc(2\theta)=\frac{1}{2}\sec\theta\csc\theta$

5 pts 3. Determine if the following equation is TRUE or FALSE. Show your work.

$$2\log_3 4-3\log_3 2=\frac{\ln 2}{\ln 3}$$

ANSWER: **TRUE** **OR** **FALSE ?**

5 pts

4. **Circle TRUE OR FALSE**

A) $\log_2(5x) = \log_2 5 + \log_2 x$ ………………………………………… **TRUE OR FALSE**

B) $\frac{\log_6(5)}{\log_6(x)} = \log_6 5 - \log_6 x$ ………………………………………… **TRUE OR FALSE**

C) $\lim_{x\to\infty} \log_{\frac{1}{3}} x \to \infty$ ………………………………………… **TRUE OR FALSE**

D) $\lim_{x\to 0^+} \log_5 x \to -\infty$ ………………………………………… **TRUE OR FALSE**

E) $\lim_{x\to 0^+} (x^2+1)^x$ is a limit whose initial form is indeterminate………………… **TRUE OR FALSE**

16 pts

5. *Find the following limits if they exist; if not, specify why they don't exist (e.g. DNE bdd.,DNE unbdd., $\to\infty$, $\to -\infty$).*
YOU MUST **SHOW** SOME **WORK and/or give some reasoning**.

A) $\lim_{x\to 0} \frac{e^x - \cos x}{\sin x}$

Answer____________

B) $\lim_{x\to 1^+} [(\ln x)^{\frac{1}{x-1}}]$

Answer___________

C) $\lim_{x\to 0} \left(\frac{1}{2x} - \frac{5\cos x}{10x} \right)$

Answer_____________

D) $\lim_{x\to 2} \frac{\sin(x-2)}{x^2 - x - 2}$

Answer___________

8pts

6. **Fill in the blanks:** A) $3^{\log_3 x} =$ ______ $\forall x \in$ ________ . B) $\forall x \neq 0,\ \frac{d}{dx}(\ln|x|) =$ ________ .

C) $\log_4 16^x =$ ______ $\forall x \in$ ________ . D) $\log_{\frac{1}{2}} 1 =$ _____ . E) If $f(x) = \log_7 x$, $f'(x) =$ _________ .

12 pts

7. Compute $f'(x)$ for each of the following functions.

A) $f(x) = \ln|\sin x|$

B) $f(x) = x^{\cos x}$

C) $f(x) = \cot(\tan(x))$

D) $f(x) = \sec^2(e^x)$

9 pts

8. A) What is the domain of the function $f(x) = \ln|\sin x|$?

B) Use 7A) and 8A) to determine all critical points of $f(x) = \ln|\sin x|$.

C) Is the statement " $f(x) = \ln|\sin x|$ is always concave down on its domain" **TRUE OR FALSE** ? *Justify with calculus!*

8 pts

9. Use ***logarithmic differentiation*** to determine a formula for $f'(x)$ if $f(x)=\left[\frac{g(x)(p(x)+r(x))\sqrt{s(x)}}{h(x)}\right]^{100}$ and p,r,s,g,h are differentiable.

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10. Complete: A) $\cot(\frac{3\pi}{2})=$ ________ B) $\sec(\frac{\pi}{6})=$ _______ C) $\tan(\frac{7\pi}{4})=$ ________ D) $\cos(-\frac{\pi}{3})=$ _______

12 pts

E) If $\sin(\theta)=\frac{1}{2}$ and $\frac{\pi}{2}<\theta<\pi$, then $\cos(\theta)=$ _______ . F) $y=\tan(x)$ has vertical asymptotes at $x=$ _____ $+k\pi,\ k\in$ Integers.

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11. A) *Determine each limit below; if it doesn't exist, specify why (e.g. DNE bdd.,DNE unbdd., $\to\infty$, $\to-\infty$).* NO work needed!

15 pts

$\lim\limits_{x\to\infty}\cos x$	$\lim\limits_{x\to\infty}\frac{\cos^2 x}{x}$	$\lim\limits_{x\to\infty}x\cos\frac{1}{x}$	$\lim\limits_{x\to\infty}x\cos x$	$\lim\limits_{x\to\infty}\frac{x^{100}}{e^x}$

B) Now determine $\lim\limits_{x\to0^+}x^{\sin x}$. *Show work this time!*

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12. Barney the virus-ologist specializes in studying the behavior of computer viruses. When Barney came to work one day, he noticed that 30 computers in his company were initially infected by an unknown virus he called the "Purple Dinosaur". However, in the end, it appeared that the virus infection would level off to about 60 infected company computers.

15 pts

A) Barney has come up with the following six functions which he thinks might be used to model the "Purple Dinosaur" virus. Which of the following functions can you **rule out** as a model for the number of computers infected by the virus based on the information above?

(a) $V(t)=\dfrac{60e^2}{e^2+e^{-0.5(t-4)}}$ (b) $V(t)=30e^{t/4}+60t$ (c) $V(t)=\dfrac{60e^{4t}}{1+e^{0.5t}}$

(d) $V(t)=30(1+.04)^t$ (e) $V(t)=30e^{-t/4}(\sin 4t+\cos 4t)+\dfrac{60t}{t+1}$ (f) $V(t)=\dfrac{60\ln(t/4+1)+30}{t/4+1}$

Answer:______________

B) On further studying the spread of this computer virus, he noticed that the number of infected computers is always increasing, although initially it is increasing at a slow rate. After four days, he observes that the population is rapidly increasing. However, after several more days, the rate of increase of the population drops. Which of Barney's functions is the best model for the number of computers infected by the virus? Justify your answer with a complete, yet concise, explanation.

Trig identities that may or may not be useful:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$