

1. Determine the interval(s) where $f(x) = x^2e^x$ is decreasing. 8 pts

ANS _____

2. Find $h'(2)$ if $h(x) = f(\frac{2}{x})$ and $f(2) = 0, f'(2) = 5, f(1) = 4, f'(1) = 3$. 4 pts

ANS _____

3. Let $f(x) = e^x$ and use the **LIMIT definition** of derivative to **derive** the formula for $f'(x)$. 12 pts
Give a brief reason after each step.
I'll start you off:

$$f'(x) \equiv \lim_{h \rightarrow 0}$$

by definition of derivative

4. A) Find all points on the graph of $4x^2 - x^3y^2 = 1$ with x -coordinate $x = 1$. 16 pts

ANSWER _____

B) Find $y'(x)$ for the curve defined by $4x^2 - x^3y^2 = 1$.

ANSWER _____

C) Find the equation of the tangent line to the curve $4x^2 - x^3y^2 = 1$ at the point(s) you found in part A).

ANSWER _____

D) Determine any points on the curve $4x^2 - x^3y^2 = 1$ where its graph has a vertical tangent.

ANSWER _____

5. Circle TRUE OR FALSE

- A) $e^{\ln(5)^x} = 5^x$ TRUE OR FALSE
- B) $3(2^x) = 6^x$ TRUE OR FALSE
- C) $7^{-x} > 0 \quad \forall x \in \mathbb{R}$ TRUE OR FALSE
- D) The range of the function $f(x) = \ln(x)$ is $(-\infty, \infty)$ TRUE OR FALSE
- E) $f(x) = x^x$ is an exponential function..... TRUE OR FALSE

6. Find the following limits if they exist. Specify, where possible, if they tend toward $+\infty$ or $-\infty$ if they don't exist. YOU MUST SHOW SOME WORK and/or give some reasoning.

18 pts

A) $\lim_{x \rightarrow -\infty} 5^x$

Answer _____

B) $\lim_{x \rightarrow 0^+} \frac{x^{\frac{8}{3}} - x^{\frac{7}{4}}}{x^2}$

Answer _____

C) $\lim_{x \rightarrow 0} 3\left(\frac{1}{4}\right)^x$

Answer _____

D) $\lim_{x \rightarrow \infty} \frac{4^x - 3^x}{5^x}$

Answer _____

E) $\lim_{x \rightarrow \infty} \left(1 + \frac{7}{x}\right)^{-3x}$

Answer _____

F) $\lim_{x \rightarrow 0^+} (1 + 5x)^{\frac{2}{x}}$

Answer _____

7. Fill in the blanks:

8 pts

A) $\lim_{x \rightarrow \infty} e^x \rightarrow \infty$ iff $\forall M > 0 \exists N > 0$ s.t. _____ \Rightarrow _____ .

B) $e^{\ln x} =$ _____ $\forall x \in$ _____ .

C) $b^x = e^{kx}$ where $k =$ _____ and $b =$ _____ .

D) Why was the fact that $f(x) = e^x$ is a 1-1 function so important? _____.

8. Compute $f'(x)$ for each of the following functions. Do NOT simplify!!!

24 pts

A) $f(x) = \frac{5^x}{x}$

B) $f(x) = (2x+1)^{\frac{1}{3}} + e^{2x}$

C) $f(x) = e^{(e^x)}$

D) $f(x) = x\sqrt{e^{(3x^2+1)}}$

E) $f(x) = 4^{1-2^x}$

F) $f(x) = xg(x) + (h(x))^4$

9. A) Find all critical points of the function $f(x) = e^{(x^3 - 3x^2 + 2)}$ and determine if a local max of f occurs at any critical point(s). 10 pts

Critical Pts: _____

Local max at: _____

10) A) Determine all horizontal asymptotes for the function $f(x) = \frac{1}{3 + 2^x}$. 6 pts

B) Every exponential function $f(x) = Ab^x$ ($b > 0, b \neq 1$) has $y = 0$ as the only horizontal asymptote. **TRUE OR FALSE ?** 4 pts

11. A paper cotton candy cone has a height of 16 cm . and a top radius of 4 cm . Little Janie finds the cone and fills it with chocolate syrup just for fun. Janie discovers that the cone, however, has a small leak at the bottom, and so she holds it over her mouth to conveniently drip the chocolate into her mouth at a rate of $2 \text{ cm}^3/\text{sec}$. (so, the amount of chocolate syrup in the cone is *decreasing*... *what does that mean about the corresponding derivative?!?.*) Recall: The volume of a cone with height h and radius r is 15 pts

$V = \frac{1}{3} \pi r^2 h$. Similar triangles may also be useful here!!

A) How fast is the height of the chocolate syrup changing at the *instant* when the height of the chocolate syrup in the cone is 8 cm ?

ANS _____

B) Give an equation that relates the rate of change of the radius of the chocolate syrup at any time to the rate of change of its height.

ANS _____

12. Dr. Drosophila is breeding fruit flies in his laboratory. He started with 600 fruit flies, and after 3 days there were already 840 fruit flies. Assume that the fruit fly population grows at a rate proportional to the number of fruit flies in the lab. 15 pts

A) Give a formula for the number of fruit flies in the lab at any time t , and determine the *exact* value of the *continuous growth rate* of the fruit fly population.

B) How long will it take for the fruit fly population to quadruple?

ANS _____

