TEST #3 **Total Points: 150** **Name**_____

Math 103 Fall 07 *Extra Credit:* Name the gifts given by the true love in the 12 Days of Christmas song for the days x that are composite numbers such that $x \not\equiv 0 \bmod 3$.

1. Circle **True** or **False** for each of the following, and complete the answer to the final question. 50 pts.

a) 13^2 is a prime number ……..…….…….……. **True** or **False**

b) There are no numbers a, b, c such that $a^4 + b^4 = c^4$ ……...……...…. **True** or **False**

c) Goldbach's conjecture proved all even numbers > 2 can be written as the sum of two primes…. **True** or **False**

d) The prime numbers have an identity element under the operation of multiplication…...…… **True** or **False**

e) A mathematical system cannot have an identity if all elements do not have an inverse…… **True** or **False**

f) $a + (b + c) = (b + c) + a$ is an example of the commutative property…….. **True** or **False**

g) Large sums of money have been paid to people able to compute million digit prime numbers... **True** or **False**.

h) The golden ratio is approximated by F_{n+1} / F_n where F_n is the n^{th} Fibonacci number……... **True** or **False**

i) The golden ratio is exactly equal to $(1+\sqrt{3})/2$ …….. **True** or **False**

j) The odd integers with the operation of multiplication satisfies the closure property……... **True** or **False**

k) The associative property holds for addition and subtraction of real numbers……...…..…….. **True** or **False**

l) For addition of natural numbers, there is no identity element……...…..…….. **True** or **False**

m) The integers form an Abelian group under addition……..... **True** or **False**

n) Every composite number is divisible by 2…….. **True** or **False**

o) A 6-hour clock is based on the ordinary clock face with numerals from the set {1, 2, …, 6}…. **True** or **False**

p) The set of all subsets of $\mathbb{N}$ with the operation of union has $\varnothing$ as an identity element……... **True** or **False**

q) Every composite number has one and only one prime factorization……...……...**True** or **False**

r) The set of Fibonacci numbers with the operation of addition satisfies the closure property…….**True** or **False**

s) All prime numbers have not been discovered yet, but there can only be finitely many primes.....**True** or **False**

t) A Mersenne prime is always produced by the formula $2^p - 1$ where p is any prime number….. .**True** or **False**

u) If the sum of the digits of a natural number is divisible by 9, then the number is divisible by 3....**True** or **False**

v) The set of even integers under the operation of multiplication satisfies the identity property…...**True** or **False**

w) If n is a natural number and 4 divides n and 2 divides n, then 8 divides n..…….....**True** or **False**

x) The set of all subsets of $\mathbb{N}$ with the operation of intersection satisfies the commutative property..**True** or **False**

y) The Lucas sequence and the Fibonacci sequence both contain the number 21……..…… **True** or **False**

2. Complete the following *distributive property:*

$$p \wedge (q \vee r) \equiv \_\_\_\_\_$$

3 pts.

3. (a) (5pts) Give the prime factorization of 2400. *Show your work.*

(b) (2pts) Determine the number of divisors of 2400.

7 pts.

Answer: 2400 = _____

4. In (a)—(e), *use divisibility tests* to determine if 15,041,208 is divisible by 6,7,8,9, or 4. ***Show your work!***

17 pts.

(a) (4pts) 6 **Yes or No (circle)**

(b) (4pts) 7 **Yes or No (circle)**

(c) (3pts) 8 **Yes or No (circle)**

(d) (3pts) 9 **Yes or No (circle)**

(e) (3pts) 4 **Yes or No (circle)**

5. For each of the following, give the next equation in the pattern and a general formula for the n^{th} equation in the pattern (let F_n represent the n^{th} Fibonacci number).

14 pts.

(a)
$$1^2 + 1^2 = 2$$
$$1^2 + 2^2 = 5$$
$$2^2 + 3^2 = 13$$

next equation: _____

general formula: _____ $n \geq$ __

(b)
$$8 - 5 + 3 = 6$$
$$13 - 8 + 5 = 10$$
$$21 - 13 + 8 = 16$$

next equation: _____

general formula: _____ $n \geq$ __

6. Use the Sieve of Eratosthenes method to determine all prime numbers in the set below. *Indicate your process by appropriate markings.*

9 pts.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
37	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**

At what value of *n* can you stop the process and be sure that you have "sifted out" all primes above? *n* = _____.

How many pairs of twin primes are there in the above table? _____ .

7. Which group properties hold and which do not for each of the following mathematical systems? *For each, give a specific example that demonstrates your conclusion.* Give the identity and inverse elements when these properties hold.

16 pts.

(a) *natural numbers* under *multiplication:*	(b) *even integers* under *addition:*
closure: Yes or No ?	**closure: Yes or No ?**
associative: Yes or No ?	**associative: Yes or No ?**
identity: Yes or No ?	**identity: Yes or No ?**
inverse: Yes or No ?	**inverse: Yes or No ?**

8. Complete each of the following.

4 pts.

(a) Determine if the following is a valid statement: $48 \equiv 3 \pmod 9$ ……. **True** or **False ?**

(b) Find the smallest positive number *b* such that $25 \equiv b \pmod{11}$. **Answer:** b=_____.

9. Find the sets of all positive integers *x* that solve each of the following equations. *List at least the first four values for x.*

8 pts.

(a) $x \equiv 2 \pmod 3$

(b) $(8x+12) \equiv 0 \pmod 4$

10. Complete each of the following.

8 pts.

(a) An Abelian group is a group that also satisfies the _____ property.

(b) The set of rational numbers under the operation of multiplication {*satisfies, fails*} the inverse property because:
(circle one!)

_____.

(c) When the system of whole numbers is extended to the system of integers, the _____ property of _____ is gained?
(operation)

(d) The notation $b \mid a$ means that there exists a natural number *k* such that _____.
(equation)

11. Complete the addition and multiplication tables below for **mod 4** arithmetic and answer the following questions.

14 pts.

(a) (3 pts.)

+	0	1	2	3
0	0	1	2	3
1	1			
2	2			
3	3			

(b) (2pts) $1-3=$ _____.

(c) (2 pts) The additive inverse of 3 is _____.

(e) (2 pts) $2 \div 3 =$ _____.

(f) (2 pts) The multiplicative inverse of 3 is _____.

(d) (3 pts.)

×	0	1	2	3
0	0	0		
1				
2				
3				