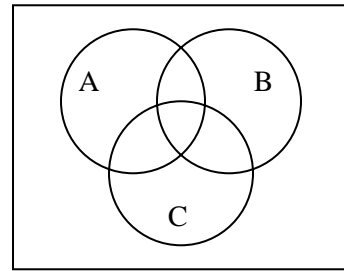


50 pts.

1. Circle **True** or **False** for each of the following: *Pay attention to notation!* (2 pts. each)
- a) $-9 \in \{\dots, -2, -1, 0, 1, 2, \dots\}$ **True** or **False**
 - b) $\{3\} \in \{2, 3, 4, 5\}$ **True** or **False**
 - c) $\{3\} \subseteq \{2, 3, 4, 5\}$ **True** or **False**
 - d) $\{1, 2, 3, \dots\} \subseteq \mathbb{N}$ **True** or **False**
 - e) $\{y, u, c, k\}$ is both equivalent to and equal to $\{u, c, k, y\}$ **True** or **False**
 - f) $\emptyset \subset \{0\}$ **True** or **False**
 - g) $\{x \mid x \text{ is a whole number and } x \text{ is not positive}\} = \emptyset$ **True** or **False**
 - h) $\emptyset \in \{\dots, -2, -1, 0, 1, 2, \dots\}$ **True** or **False**
 - i) It is not possible for the truth table of a compound logical statement to have exactly 20 rows.... **True** or **False**
 - j) $\{\emptyset\} \subseteq \emptyset$ **True** or **False**
 - k) $52 \in \{1, 2, 4, 8, \dots\}$ **True** or **False**
 - l) There are exactly 15 proper subsets of the set $\{2, 3, 4, 5\}$ **True** or **False**
 - m) $\{(1, 2), (3, 4)\} = \{(3, 4), (1, 2)\}$ **True** or **False**
 - n) If $n(A) = 4$ and $n(A \times B) = 8$, then $n(B) = 4$ **True** or **False**
 - o) $\{(-1, 1), (-2, 2)\} = \{(1, -1), (2, -2)\}$ **True** or **False**
 - p) $A \cap A' = \emptyset$ **True** or **False**
 - q) $A \cap B \subseteq A$ **True** or **False**
 - r) For A, B finite sets, $n(A \cup B) = n(A) + n(B)$ **True** or **False**
 - s) If $n(A) = \aleph_0$ and $n(B) = \aleph_0$, then $n(A \cup B) = \aleph_0$ **True** or **False**
 - t) There are more irrational numbers than there are rational numbers..... **True** or **False**
 - u) There exists an integer that is not a rational number..... **True** or **False**
 - v) There are more rational numbers than there are positive integers..... **True** or **False**
 - w) The set of even integers $= \{\dots -4, -2, 0, 2, 4, \dots\}$, and this set is countable..... **True** or **False**
 - x) $p \wedge \sim p$ is always FALSE, but $p \vee \sim p$ is always TRUE..... **True** or **False**
 - y) $\{x \in \mathbb{R} \mid 0 < x < 1\}$ is an uncountably infinite set..... **True** or **False**

2. (a) (8 pts) Use the Venn Diagram to shade the set $A \cap (B' \cup C)$.
 Be sure your final answer is clearly shaded!



10 pts.

(b) (2 pts) Does $(A \cap B') \cup C = A \cap (B' \cup C)$? **Yes** or **No**?
 (circle one!)

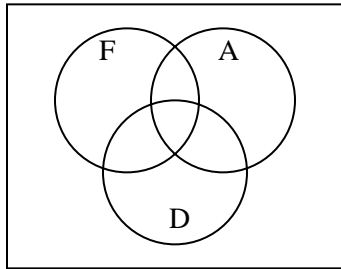
3. Old MacDonald had a farm, eey-ie, eey-ie, oh! And on that farm he had some chickens, eey-ie, eey-ie, oh!
 With 21 pts.

F=the set of Free range chickens,
 A=the set of Adorable chickens, and
 D=the set of chickens scheduled to be Dinner.

and

$n(F) = 37, n(F \cap A' \cap D) = 9, n(F \cap A \cap D) = 2,$
 $n(F \cap A) = 11, n(F' \cap A' \cap D') = 18,$
 $n(A') = 50, n(F' \cap A \cap D) = 5, n(A \cap D') = 16.$

Where indicated, also identify the corresponding set using F, A, D , intersection, and/or complement.



- (a) How many free range chickens were about to be dinner? _____ Set: _____
- (b) How many free range chickens were not adorable?..... _____ Set: _____
- (c) How many chickens had exactly one of the above attributes?..... _____
- (d) How many chickens did Old MacDonald have ?..... _____
- (e) How many adorable chickens were neither free range nor about to be dinner?..... _____ Set: _____

4. Complete each *definition* using set builder notation. (A and B are generic sets and U is the universal set).

(a) $A \cup B =$ _____ .

9 pts

(b) $B' =$ _____ .

(c) The set of rational numbers, $\mathbb{Q} =$ _____

5. (a) Write the De Morgan's Law equation that corresponds to the following statement:
The complement of the union of two sets is equal to the intersection of the complements.

6 pts

(b) Write the De Morgan's Law equation for the negation of a disjunction:

6. Circle **all correct negations** for each statement in (a) and (b).

14 pts.

(a) (5 pts) Everyone is asleep.

Everyone is not asleep. Nobody is asleep. Someone is not asleep. Not everyone is asleep. At least one is awake.

(b) (4 pts) Some ring tones are cool.

Some ring tones are not cool. No ring tones are cool. All ring tones are not cool. Not all ring tones are cool.

(c) (5 pts) Circle all existential quantifiers from the list below:

at least one every some there exists all each none

7. (a) (6 pts) Put the set A (given below) into a 1-1 correspondence with the natural numbers. Give (in the box) a general formula involving n that indicates the number in the set A that is paired with the generic natural number n . 8 pts.

$A = \{1, 3, 5, 7, \dots, \boxed{}, \dots\}$ Describe A in words: _____.

(b) (2 pt) What is the 2134th element in the set A ? _____. What is the cardinality of A ? _____.

8. (a) (5 pts) Put the set A (given below) into a 1-1 correspondence with a proper subset of itself, and give general formulas involving n that describes the pairing. 7 pts.

$A = \{1, \frac{4}{3}, \frac{5}{3}, 2, \dots, \boxed{}, \dots\}$

(b) (2 pts) Complete: By definition, part (a) shows that the set A is _____.

9. Are the following two statements equivalent logical statements? **Yes** or **No** ? Briefly justify. 7 pts.

(a) $(p \wedge \sim q) \wedge q$ (b) $\sim [(q \vee p) \vee \sim q]$

10. (a) (10 pts) Systematically construct the truth table for $r \wedge [(p \vee q) \wedge (\sim q)]$. 12 pts.

(b) (2 pts) Complete: The truth table for $\sim q \wedge [(p \vee t) \wedge (\sim s \vee r)] \vee (q \vee s)$ has _____ rows.

11. Continue Cantor's ingenious scheme to determine what natural number is paired with $\frac{1}{5}$ in establishing a 1-1 correspondence between the set of non-negative rational numbers and \mathbb{N} .

Answer: $\frac{1}{5} \leftrightarrow$ _____.

0	1	2	3	4	5	6	7	8	9	...
1	1	2	3	4	5	6	7	8	9	...
	1	2	3	4	5	6	7	8	9	...
	2	2	2	2	2	2	2	2	2	...
	1	2	3	4	5	6	7	8	9	...
↓	3	3	3	3	3	3	3	3	3	...
	1	2	3	4	5	6	7	8	9	...
	4	4	4	4	4	4	4	4	4	...
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

6 pts.