Abstract Algebra II **Van Wyk**
Extra Credit Problems **Due Whenever**

WRITE YOUR PROOFS IN COMPLETE SENTENCES. READ THE SYLLABUS REGARDING THE RESTRICTIONS ON CORRESPONDENCE AMONG STUDENTS.

1. *(10 points)* If $f : X \to Y$ is a function and $y \in Y$, then

$$f^{-1}(y) = \{x \in X \mid f(x) = y\}.$$

Notice that $f^{-1}(y)$ is a *set* which generally contains more than one element (unless f is an injection). In particular, f^{-1} is generally not a function.

Let $\phi : G \to H$ be a group homomorphism, and let $x, y \in H$. Determine what, if anything[1], the two sets $\phi^{-1}(xy)$ and $\phi^{-1}(x)\phi^{-1}(y)$ have in common[2].

(In other words, in what sense do inverse images of elements under a homomorphism imitate the homomorphism property?)

2. *(10 points)* Let X and Y be sets. If there exist surjections $f : X \to Y$ and $g : Y \to X$, then there exists a bijection $h : X \to Y$.

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[1]That is, $\subseteq$, $\supseteq$, $=$, or none of these.

[2]If $A, B \subseteq G$, then $AB = \{ab \in G \mid a \in A \text{ and } b \in B\}$.