TEST II

Math 245
April 6, 2006

Name: ____________________________________

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Your work will be graded for clarity as well as for mathematical accuracy.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 3 multi-part problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*40 pts*) Give examples of each of the following, if possible. Don't prove anything, just describe specific examples. If such an example does not exist, write "not possible".

(**a**) A relation on $\{1, 2, 3\}$ that is reflexive, symmetric, and antisymmetric.

(**b**) A relation on $\mathbb{Z}$ that is not reflexive and not symmetric.

(**c**) A relation on $\{a, b, c, d\}$ that contains (c, c) but is not reflexive.

(**d**) A function $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$.

(**e**) A function that is onto but not one-to-one.

(**f**) Functions f and g that are invertible, but whose composition $f \circ g$ is not invertible.

(**g**) A partially ordered set (give the set AND the partial order) that is not totally ordered.

(**h**) A Hasse diagram showing a partially ordered set with exactly three maximal elements.

2. (*30 pts*) Many of the proofs we have done in this class follow a similar format each time. In class we've called the outlines for such proofs "skeleton proofs;" for example, here is a skeleton proof for showing that a function $f\colon X \to Y$ is onto:

> *Proof.* Suppose $f\colon X \to Y$ is a function.
> Given any $y \in Y$, consider $x =$ _____ $\in X$.
> Then $f(x) = f($_____$) = ... = ... = y$.
> Therefore f is onto. ■

(a) Give a skeleton proof for showing that a relation $\mathcal{R}$ on a set A is symmetric:

(b) Give a skeleton proof for showing that a relation $\mathcal{R}$ on a set A is antisymmetric:

(c) Give a skeleton proof for showing that a function $f\colon X \to Y$ is one-to-one:

3. (*30 pts*) Calculate each of the following. Leave your answers unsimplified (i.e. do NOT write out factorial expressions or try to find decimal expansions).

(**a**) Find the probability of being dealt a full house (three cards of one rank and two of another rank) from a standard deck.

(**b**) You have 18 sandwiches for a picnic: 6 peanut-butter, 5 tunafish, 4 cream cheese, and 3 olive-loaf. How many distinguishable ways may these sandwiches be distributed to 18 people so that each person gets exactly one sandwich?

(**c**) If you had the same 18 sandwiches as described above, how many distinguishable ways may these sandwiches be distributed to 18 people if there is no restriction on how many (or few) each person receives?

Survey Questions: *(2 extra credit points)*

Name two questions or topics that could have been on this test, but were not.

How do you think you did?

SPACE FOR SCRAP WORK