TEST III

Math 245
December 1, 2005

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Your work will be graded for clarity as well as for mathematical accuracy. Make sure that your logic and arguments are clear. Provide reasons for steps whenever possible.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 5 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. Find the probability of being dealt a full house (three of a kind and one pair) in a 5-card poker game. Briefly justify your answer (explain how you are counting).

2. Carefully explain why for $n \in \mathbb{N}$, the number of ways that you can build a tower of height n out of blocks that have either height 1 or height 2 is equal to the $(n+1)^{\text{st}}$ Fibonacci number F_{n+1}.

3. Consider the following quasiorder and induced equivalence relation and partial order on equivalence classes:

- Let $\preceq$ be the quasiorder on the set $A = \{1, 2, \ldots, 20\}$ defined by $m \preceq n$ if and only if the highest power of 2 that divides m also divides n and the highest power of 3 that divides m also divides n.
- Let $\sim$ be the equivalence relation on A defined by $m \sim n$ if and only if $m \preceq n$ and $n \preceq m$.
- Finally, let $\leq$ be the partial order on the equivalence classes of A defined by $[a] \leq [b]$ if and only if $a \preceq b$.

Deep breath. You aren't going to have to write any proofs. Just list the elements of each equivalence class of the induced equivalence relation $\sim$, and draw the Hasse diagram for the induced partial order $\leq$ on the equivalence classes.

4. Exhibit a bijection that shows that $|[0,1]| = |[2,7]|$.

5. How many simple graphs have 5 vertices and 7 edges? Briefly justify your answer (explain how you are counting).

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK