TEST II

Math 245
October 27, 2005

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Your work will be graded for clarity as well as for mathematical accuracy. Make sure that your logic and arguments are clear. Provide reasons for steps whenever possible.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 4 multi-part problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*28 pts*) Definitions. Please give real mathematical definitions as you might see in a textbook, NOT just intuitive descriptions.

(IMPORTANT NOTE: Part (b) actually involves two properties that you must define. Parts (c) and (d) involve three properties that you must define. Don't just name the properties; be sure to name them AND define them.)

(**a**) A relation R from A to B is a *function* if:

(**b**) A function $f\colon A \to B$ is a *bijection* if:

(**c**) A relation $\sim$ on a set A is an *equivalence relation* if:

(**d**) A relation $\leq$ on a set P is a *partial order* if:

2. (*24 pts*) Counting and calculations. Please circle your final answers.

(**a**) At the Pizza Palace you can order a large pizza with up to three *distinct* toppings for $9.99. There are 12 toppings to choose from. How many different pizzas are possible?

(**b**) An account number consists of seven digits from $0, 1, 2, \ldots, 9$. Digits *can* be repeated. How many account numbers contain the number 6 exactly three times?

(**c**) How many distinguishable rearrangments are there of the letters of the word BOONDOGGLE?

(**d**) A teacher has 12 gold stars and 35 silver stars to give to 20 students. If each student gets at least one silver star, how many different ways can the teacher distribute the stars?

3. (*24 pts*) Proofs. Style counts. Feel free to write drafts on the scrap page first to collect your thoughts. PLEASE, NO RAMBLING! BE CONCISE!

(a) Argue that $\binom{n}{k} = \binom{n}{n-k}$ in such a way that your argument uses the words "choose" and "subset" and "ways."

(b) Prove that the relation R on $\mathbb{Z}$ defined by $x\,R\,y \iff xy > 0$ is symmetric.

(c) Prove that the function $f\colon [2,4] \to [0,2]$ defined by $f(x) = \sqrt{x-2}$ is one-to-one.

(d) Prove that if $f\colon A \to B$ and $g\colon B \to C$ are onto, then $g \circ f\colon A \to C$ is onto.

4. (*24 pts*) Examples and counterexamples.

(a) Give an example of a relation on the set $A = \{a, b, c, d\}$ that is reflexive but not symmetric.

(b) Consider the relation defined by $A \# B \iff A \cap B = \emptyset$, for all $A, B \in \mathcal{P}(\{1, 2, 3\})$. Give a counterexample that shows that this relation is not transitive.

(c) Suppose $(P_1, \leq_1)$ and $(P_2, \leq_2)$ are totally ordered spaces. Must $P_1 \times P_2$ with the product order be totally ordered? The answer is no; give a counterexample.

(d) Give an example of a set X and a subset S with the property that S has upper bounds in X, but no least upper bound in X.

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK