TEST I

Math 245
September 29, 2005

Name: _____
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Your work will be graded for clarity as well as for mathematical accuracy. Make sure that your logic and arguments are clear. Provide reasons for steps whenever possible.
- Don't get hung up on any one problem. If you get stuck, move on and come back to the problem later.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 4 multi-part problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*24 pts*) Determine whether each of the following statements is true (T) or false (F). You do not have to provide reasons or counterexamples.

(**a**) **T** **F** $\forall n \in \mathbb{N}, \exists m \in \mathbb{N}$ such that $m > n$.

(**b**) **T** **F** $\exists m \in \mathbb{N}$ such that $\forall n \in \mathbb{N}, mn = n$.

(**c**) **T** **F** If S and T are nonempty finite sets, then $|S \times T| = |S| \cdot |T|$.

(**d**) **T** **F** If X and Y are sets with $|X| = 43$, $|Y| = 32$, and $|X \cap Y| = 7$, then $|X \cup Y| = 75$.

(**e**) **T** **F** If A and B are statements, then $\neg(A \vee B) = \neg A \vee \neg B$.

(**f**) **T** **F** If A, B, and C are statements, then $\neg(A \Rightarrow B) \iff (A \wedge \neg B)$.

(**g**) **T** **F** If A, B, and C are statements, then $(A \wedge B) \vee C = (A \vee C) \wedge (B \vee C)$.

(**h**) **T** **F** If $\{S_i\}_{i \in I}$ is a collection of sets, then $\left(\bigcap_{i \in I} S_i \right)^c = \bigcap_{i \in I} (S_i)^c$.

2. (*12 pts*) Short answers.

(**a**) Suppose S_n is a statement involving $n \in \mathbb{N}$. If S_1 is true, and if the implication $(S_k \implies S_{k+2})$ holds for every $k \in \mathbb{N}$, then you can conclude that S_n is true for which values of n?

(**b**) State the strong form of the pigeonhole principle. (Your statement should be written well enough to appear as a theorem in a textbook; you may want to write a draft on the scrap page before writing your answer here.)

3. (*28 pts*) Consider the statement:

"The square of any odd integer is odd."

In this problem you will consider various methods of proving this statement. For parts (a), (b), and (c) you will only write down the first and last lines of the proof. In part (d) you will actually prove the statement (style counts, so you may want to write a draft on the scrap page first!).

a. (*6 pts*) To prove the statement directly,

ASSUME:

SHOW:

b. (*6 pts*) To prove the statement by contrapositive,

ASSUME:

SHOW:

c. (*6 pts*) To prove the statement by contradiction,

ASSUME:

SHOW:

d. (*10 pts*) Prove the statement directly.

(Example of what I mean for parts (a)-(c), using a different statement: If A and B are sets, then to prove $A \subseteq B$, you could ASSUME that $x \in A$ and SHOW that $x \in B$.)

4. (*36 pts*) More short answers.

(**a**) If $S = \{x, y, z\}$ then the power set of S is:

(**b**) If $S = \{x, y, z\}$ then list all of the possible partitions of S:

(**c**) Find a condition on x that is sufficient but not necessary to make x a solution to $x^2 < 4$.

(**d**) Find a condition on x that is necessary but not sufficient to make x a solution to $x^2 < 4$.

(**e**) Negate the following statement: $\forall \epsilon > 0, \exists \delta > 0$ such that $0 < |x - 1| < \delta \Rightarrow |\frac{3}{x+2} - 1| < \epsilon$.

(**f**) Write the following in LaTeX:

The sets in a collection $\mathcal{C} = \{S_i \mid i \in I\}$ are *mutually disjoint* if $\forall S_i, S_j \in \mathcal{C}$, $S_i \neq S_j \Rightarrow S_i \cap S_j = \emptyset$.

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK