Infinite Series **Math 236 Summer 2002**

This worksheet is a review of the types and techniques of analyzing infinite series. You do not have to hand in this worksheet. At least two of these problems will be on the final.

1. Express $\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \ldots + \frac{1}{90}$ in sigma notation.

2. Express $\frac{3^3}{e^2} + \frac{3^2}{e^3} + \ldots + \frac{1}{3^2 e^7}$ in sigma notation:

(**a**) Starting from $k = 3$.

(**b**) Starting from $k = 2$.

(**c**) Starting from $k = 0$.

3. Show that the n^{th} partial sum for the geometric series $\sum_{k=0}^{\infty} x^k$ is $s_n = \frac{1 - x^{n+1}}{1 - x}$.

4. Find a formula for the n^{th} partial sum s_n for the series $\sum_{k=1}^{\infty} \frac{1}{k(k+2)}$.

5. Determine whether the following series converge or diverge. If convergent, find the sum.

(**a**) $\sum_{k=1}^{\infty} \frac{3}{e^k}$

(**b**) $\sum_{k=1}^{\infty} \frac{2}{(2k-1)(2k+1)}$

(**c**) $1 - \frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \ldots$

(**d**) $\sum_{k=1}^{\infty} \frac{1}{k^2 + 3k + 2}$

6. Consider the rational number $\alpha = 0.251251251251251\ldots$

(**a**) Write the number α as an infinite series.

(**b**) Find the sum of the series and use it to express α as the quotient of two integers.

7. Find a series expansion for $\frac{x}{1 + x^3}$, $|x| < 1$.

8. Determine whether the following series converge or diverge. Justify your answers.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2k}{1+k^4}$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{2k+100}\right)^k$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+3)(k+4)}$

(d) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$

(e) $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{e^k}$

(f) $\displaystyle\sum_{k=6}^{\infty} \frac{1}{2+3^{-k}}$

(g) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$

(h) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{10^{4k}}$

(i) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\cosh^2 k}$

(j) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{2k+1}$

(k) $\displaystyle\sum_{k=3}^{\infty} \frac{10^k}{k!}$

(l) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k^2-1}}$

(m) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{3^k-2}$

(n) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k}$

(o) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\sqrt{2k^2+1}}$

(p) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k}{2k+1}\right)^k$

(q) $\displaystyle\sum_{k=0}^{\infty} \frac{3^{2k}}{(2k)!}$

(r) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k^k}$

9. Determine whether each series converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+2}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k+4}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k+\sqrt{k}}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}2^k}{3^k+1}$

(f) $\displaystyle\sum_{k=5}^{\infty} \frac{(-1)^{k+1}2^k}{k^2}$

10. Find the smallest integer n so that s_n will approximate $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{k^2+2}$ to within 0.01.