Math 220 : Elementary Statistics — Kane Nashimoto
Final Exam Solutions
Summer 2005

1. Binomial random variable.

   Note : X, number of rats turning right, is binomial with $n = 14$ and $\pi = .80$.

   (a) $P(X = 10) = P(X \le 10) - P(X \le 9) = .3018 - .1298 = .1720$

   (b) $P(X = 14) = 1 - P(X \le 13) = 1 - .9560 = .0440$ or
   $P(X = 14) = (.80)^{14} = .0440$

2. Probability.

   E : {person uses treadmill}
   F : {person uses swimming pool}

   Note : $P(E) = .70$; $P(F) = .45$; $P(E \cap F) = .35$

   (a) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = .70 + .45 - .35 = .800$

   (b) $P(F|E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{.35}{.70} = .500$

   (c) $P(E \cap F) = .35$ but $P(E)P(F) = (.70)(.45) = .315$
   $\Rightarrow$ Two events are not independent.

3. Bias in scientific studies.

   An example of a study in which responses are not obtained from all participants.

4. Numerical summary of data.

   Note : $\sum x = 15$; $\sum x^2 = 61$; $n = 7$

   (a) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{15}{7} = 2.143$

   (b) $s = +\sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n-1}} = +\sqrt{\dfrac{61 - (15)^2/7}{7-1}} = 2.193$

5. Inference about mean with σ unknown.

   (a) $\bar{x} \pm t_{.05/2,\,30-1}\dfrac{s}{\sqrt{n}}$; $32.7 \pm 2.045\dfrac{3.8}{\sqrt{30}}$; 32.7 ± 1.419 ; $(31.281\,,\ 34.119)$

   (b) $H_0 : \mu = 34.0$ vs. $H_a : \mu < 34.0$

   $t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{32.7 - 34.0}{3.8/\sqrt{30}} = -1.874$

   Critical value : -1.699 $(t_{.05,\,30-1} = 1.699)$

   Reject H_0. $(\mu < 34.0)$

6. Normal random variable.

   Note: X, clinometer's reading, is normal with $\mu = 35.00$ and $\sigma = 0.04$.

   $$P(X < 34.95) = P\left(\frac{X-\mu}{\sigma} < \frac{34.95 - 35.00}{0.04}\right) = P(Z < -1.25) = .1056$$

7. Categorical data analysis.

   (a) Chi-square goodness-of-fit test.
   $H_0: \pi_1 = .50\,;\ \pi_2 = .50$ (i. e., $\pi_1 = \pi_2$)
   $H_a: \pi_i \neq \pi_{0i}$ (i. e., $\pi_1 \neq \pi_2$)
   $E_i = n\pi_{0i}\,;\ n = 280$
   $E_1 = (280)(.50) = 140.0 \quad E_2 = (280)(.50) = 140.0$
   $$h^* = \sum_i \frac{(O_i - E_i)^2}{E_i} = \frac{(152 - 140.0)^2}{140.0} + \frac{(128 - 140.0)^2}{140.0} = 2.057$$
   Critical value: 3.841 ($h_{.05,\,2-1} = 3.841$)
   Retain H_0.
   (Insufficient evidence to conclude that the proportions of male and female flies are significantly different.)

   (b) Chi-square test of homogeneity.
   H_0: Male-to-female proportions are homogeneous for the two species.
   H_a: Male-to-female proportions are not homogeneous for the two species.

   Observed and marginal counts:

152	128	280
103	117	220
255	245	500

   $E_{ij} = (O_{i\cdot})(O_{\cdot j})/n$
   $E_{11} = (280)(255)/500 = 142.8 \quad E_{12} = (280)(245)/500 = 137.2$
   $E_{21} = (220)(255)/500 = 112.2 \quad E_{22} = (220)(245)/500 = 107.8$
   $$h^* = \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$
   $$= \frac{(152 - 142.8)^2}{142.8} + \frac{(128 - 137.2)^2}{137.2} + \frac{(103 - 112.2)^2}{112.2} + \frac{(117 - 107.8)^2}{107.8} = 2.749$$
   Critical value: 2.706 ($h_{.10,\,(2-1)(2-1)} = 2.706$)
   Reject H_0.
   (Species A has significantly more male files than female flies; Species B has significantly more female files than male flies.)

8. Independent-samples t test by SPSS.

   $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 > 0$ (1 = "Women"; 2 = "Men")
   $t^* = 1.262$ (equal variances assumed)
   one-sided p-value $= .224 \div 2 = .112$ ($> \alpha = .05$)
   Retain H_0.
   Insufficient evidence to conclude that women have significantly higher pulse rates.