Math 220 : Elementary Statistics — Kane Nashimoto

Exam 3 Solutions

Summer 2005

1. Sampling distribution.

   (a) $\bar{X}$, mean lifetime, is approximately normal with $\mu_{\bar{X}} = \mu = 750$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 750/\sqrt{50}$. This is due to the central limit theorem.

   (b) $P(\bar{X} > 1000) = P\left(\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \dfrac{1000-750}{750/\sqrt{50}}\right) = P(Z > 2.36)$

   $= 1 - P(Z \leq 2.36) = 1 - .9909 = .0091$

2. Inference about proportion.

   (a) Let $\pi = .5$.

   $n = \pi(1-\pi)\left(\dfrac{z_{.05/2}}{B}\right)^2 = (.5)(1-.5)\left(\dfrac{1.960}{.06}\right)^2 = 266.78 \;\Rightarrow\; 267$

   (b) $p = 195/250 = .780$

   $p \pm z_{.05/2}\sqrt{\dfrac{p(1-p)}{n}}$; $.780 \pm 1.960\sqrt{\dfrac{(.780)(1-.780)}{250}}$; $.780 \pm .051$; $(.729\,,\ .831)$

3. Inference about mean with σ known.

   (a) $H_0\colon \mu = 7.50$ vs. $H_a\colon \mu \neq 7.50$

   $z^* = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \dfrac{7.52-7.50}{0.10/\sqrt{50}} = 1.414$

   Critical values : ± 1.645 $(z_{.10/2} = 1.645)$

   Retain H_0. $(\mu \approx 7.50)$

   (b) p-value $= 2P(Z \geq 1.41) = 2(1 - P(Z < 1.41)) = 2(1 - .9207) = 2(.0793) = .1586$

   (Note that this p-value is greater than $\alpha = .10$.)

4. Inference about mean with σ unknown.

   (a) $\bar{x} \pm t_{.01/2,\,24-1}\dfrac{s}{\sqrt{n}}$; $213 \pm 2.807\dfrac{26}{\sqrt{24}}$; 213 ± 14.897 ; $(198.103\,,\ 227.897)$

   (b) We are 99% certain that the true mean response time will fall between 198.1 and 227.9 seconds.

   (c) The 95% confidence interval will be narrower.