Math 220 : Elementary Statistics — Kane Nashimoto

Final Exam Solutions

Summer 2009

1. Binomial random variable.

   Note : $X$, number of customers turning right, is binomial with $n = 12$ and $p = .90$.

   (a) $P(X = 10) = P(X \le 10) - P(X \le 9) = .3410 - .1109 = .2301$

   (b) $P(X < 7) = P(X \le 6) = .0005$

   (c) $P(X > 5) = 1 - P(X \le 5) = 1 - .0001 = .9999$

2. Numerical summary of data.

   Note : $\sum x = 14.61$ ; $\sum x^2 = 43.9127$ ; $n = 5$

   $$\bar{x} = \frac{\sum x}{n} = \frac{14.61}{5} = 2.922$$

   $$s = +\sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}} = +\sqrt{\frac{43.9127 - (14.61)^2/5}{5-1}} = 0.553$$

3. Inference about mean.

   $H_0 : \mu = 649.99$ vs. $H_a : \mu < 649.99$

   $$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{643.39 - 649.99}{17.71/\sqrt{60}} = -2.887$$

   $-t_{.005,\,60-1} = -2.662 > t^* = -2.887 > -t_{.0025,\,60-1} = -2.916$

   $.0025 < p\text{-value} < .005$

   Reject $H_0$ at $\alpha = .05$. ($\mu < 649.99$)

4. Probability.

   $A$: {health conscious}

   $B$: {organic vegetable}

   Note : $P(A) = .67$ ; $P(B) = .42$ ; $P(A \cap B) = .31$

   (a) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.31}{.67} = .463$

   (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .67 + .42 - .31 = .780$

   (c) $P(A \cap B) = .31$ but $P(A)P(B) = (.67)(.42) = .281$

   $\Rightarrow$ Two events are not independent.

5. Inference about proportion.

   (a) $\hat{p} = 76/271 = .280$

   $\hat{p} \pm z_{.01/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ ; $.280 \pm 2.576\sqrt{\dfrac{(.280)(1-.280)}{271}}$ ; $.280 \pm .070$ ; $(.210\,,\ .350)$

   (b) Use $\hat{p} = .280$.

   $$n = \frac{\hat{p}(1-\hat{p})\, z^2_{.10/2}}{m^2} = \frac{.280(1-.280)(1.645)^2}{(.05)^2} = 218.21 \;\Rightarrow\; 219$$

6. Normal random variable.

   Note: $X$, scale reading, is normal with $\mu = 100.32$ and $\sigma = 0.04$.

   (a) $P(X < 100.35) = P\left(\frac{X-\mu}{\sigma} < \frac{100.35 - 100.32}{0.04}\right) = P(Z < 0.75) = .7734$

   (b) $z = -0.25 \because P(Z \leq -0.25) \approx .40$

   $z = \frac{x-\mu}{\sigma}$; $-0.25 = \frac{x - 100.32}{0.04}$; $x = 100.32 + (-0.25)(0.04) = 100.31$

7. Experimental design.

   An example of a study in which two explanatory variables both affect the response variable and, hence, the true cause of the response is unclear.

8. Chi-square test of independence.

   $H_0$: Sodium content and price are independent.
   $H_a$: Sodium content and price are not independent.

   Observed and marginal counts:

| | | |
|---|---|---|
| 11 | 24 | 35 |
| 39 | 29 | 68 |
| 50 | 53 | 103 |

   $\widehat{n_{ij}} = (n_{i\cdot})(n_{\cdot j})/n$
   $\widehat{n_{11}} = (35)(50)/103 = 16.99$ $\widehat{n_{12}} = (35)(53)/103 = 18.01$
   $\widehat{n_{21}} = (68)(50)/103 = 33.01$ $\widehat{n_{22}} = (68)(53)/103 = 34.99$

   $$h^* = \sum_i \sum_j \frac{(n_{ij} - \widehat{n_{ij}})^2}{\widehat{n_{ij}}}$$
   $$= \frac{(11-16.99)^2}{16.99} + \frac{(24-18.01)^2}{18.01} + \frac{(39-33.01)^2}{33.01} + \frac{(29-34.99)^2}{34.99} = 6.217$$

   $h_{.025,\,(2-1)(2-1)} = 5.024 < h^* = 6.217 < h_{.010,\,(2-1)(2-1)} = 6.635$
   $.010 < p\text{-value} < .025$
   Reject $H_0$ at $\alpha = .10$.
   (Sodium content and price have an inverse relationship.)

9. Paired-samples $t$ test by SPSS.

   $H_0: \mu_\delta = 0$ vs. $H_a: \mu_\delta > 0$ ("respondent's height" minus "father's heigh")
   $t^* = 1.372$
   one-sided $p$-value $= .189 \div 2 = .0945$
   Retain $H_0$ at $\alpha = .05$.
   Insufficient evidence to conclude that the mean height for the respondents is significantly higher than that for their fathers.