Math 220 : Elementary Statistics — Kane Nashimoto

Exam 3 Solutions

Summer 2009

1. Confidence interval for proportion.

   (a) $\hat{p} = 137/300 = .457$

   $\hat{p} \pm z_{.10/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$; $.457 \pm 1.645\sqrt{\dfrac{(.457)(1-.457)}{300}}$; $.457 \pm .047$; $(.410\,,\ .504)$

   (b) Use $\hat{p} = .457$.

   $$n = \frac{\hat{p}(1-\hat{p})\, z^2_{.01/2}}{m^2} = \frac{.457(1-.457)(2.576)^2}{(.08)^2} = 257.29 \ \Rightarrow\ 258$$

2. Sampling distribution of mean.

   Note : $\bar{X}$, mean work hour, is approximately normal by the central limit theorem with $\mu_{\bar{X}} = \mu = 6.95$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 0.21/\sqrt{70}$.

   $$P(\bar{X} < 7.0) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{7.0-6.95}{0.21/\sqrt{70}}\right) = P(Z < 1.99) = .9767$$

3. Test of hypotheses about mean.

   $H_0 \colon \mu = 13.05$ vs. $H_a \colon \mu < 13.05$

   $$t^* = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{13.047-13.05}{0.013/\sqrt{25}} = -1.15$$

   $-t_{.15,\,25-1} = -1.059 > t^* = -1.15 > -t_{.10,\,25-1} = -1.318$

   $.10 < p\text{-value} < .15$

   Retain H_0 at $\alpha = .01$. ($\mu \approx 13.05$)

4. Test of hypotheses about proportion.

   $H_0 \colon p = .5$ vs. $H_a \colon p > .5$

   $\hat{p} = 89/150 = .593$

   $$z^* = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{.593-.5}{\sqrt{(.5)(1-.5)/150}} = 2.278$$

   p-value $= P(Z \geq 2.28) = 1 - P(Z < 2.28) = 1 - .9887 = .0113$

   Reject H_0 at $\alpha = .10$. ($p > .5$)

5. Confidence interval for mean.

   (a) $\bar{x} \pm t_{.05/2,\,45-1}\dfrac{s}{\sqrt{n}}$; $37.7 \pm 2.015\dfrac{2.9}{\sqrt{45}}$; 37.7 ± 0.871 ; $(36.829\,,\ 38.571)$

   (b) $n = \dfrac{4s^2}{m^2} = \dfrac{4(2.9)^2}{(0.6)^2} = 93.44 \ \Rightarrow\ 94$

6. Concept of significance test.

   (a) $H_0 \colon \mu = 78$ was rejected in favor of $H_a \colon \mu > 78$, leading to the conclusion that the average test score was significantly higher than 78.

   (b) Reject H_0. If a test rejects H_0 at $\alpha = .05$, it will reject H_0 at any $\alpha > .05$.