Math 220 : Elementary Statistics — Kane Nashimoto

Final Exam Solutions

Spring 2009

1. Inference about mean.

   (a) $\bar{x} \pm t_{.10/2,\,43-1}\dfrac{s}{\sqrt{n}}$; $1.91 \pm 1.682\dfrac{0.07}{\sqrt{43}}$; $1.91 \pm .018$; $(1.892\,,\ 1.928)$

   (b) $n = \dfrac{4s^2}{m^2} = \dfrac{4(0.07)^2}{(0.015)^2} = 87.11 \ \Rightarrow\ 88$

2. Numerical summary of data.

   Note : $\sum x = 106$; $\sum x^2 = 1976$; $n = 6$

   $\bar{x} = \dfrac{\sum x}{n} = \dfrac{106}{6} = 17.667$

   $s = +\sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n-1}} = +\sqrt{\dfrac{1976 - (106)^2/6}{6-1}} = 4.546$

3. Inference about proportion.

   $H_0 : p = .25$ vs. $H_a : p > .25$

   $\hat{p} = 131/458 = .286$

   $z^* = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \dfrac{.286 - .25}{\sqrt{(.25)(1-.25)/458}} = 1.781$

   p-value $= P(Z \geq 1.78) = 1 - P(Z < 1.78) = 1 - .9625 = .0375$

   Reject H_0 at $\alpha = .10$. $(p > .25)$

4. Binomial random variable.

   Note : X, number of red lights, is binomial with $n = 7$ and $p = .60$.

   (a) $\mu = np = (7)(.60) = 4.20$

   (b) $P(X = 3) = P(X \leq 3) - P(X \leq 2) = .2898 - .0963 = .1935$

   (c) $P(X > 5) = 1 - P(X \leq 5) = 1 - .8414 = .1586$

5. Probability.

   A : {Selected customer is a woman}

   B : {Selected customer uses self-checkout lane}

   Note : $P(A) = .68$; $P(B) = .31$; $P(A \cap B) = .18$

   (a) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.18}{.68} = .265$

   (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .68 + .31 - .18 = .81$

   (c) $P(A \cap B) = .18$ but $P(A)P(B) = (.68)(.31) = .211$

   $\Rightarrow$ Two events are not independent.

6. Normal random variable.

   Note: X, aptitude test score, is normal with $\mu = 650$ and $\sigma = 100$.

   (a) $P(X < 525) = P\left(\frac{X-\mu}{\sigma} < \frac{525-650}{100}\right) = P(Z < -1.25) = .1056$

   (b) $z = 0.39 \;\because\; P(Z \le 0.39) \approx .65$

   $z = \frac{x-\mu}{\sigma}$; $0.39 = \frac{x-650}{100}$; $x = 650 + (0.39)(100) = 689$

7. Probability.

   $P(\text{at least one revocation}) = 1 - P(\text{all no revocation}) = 1 - (.87)^4 = .427$

8. Chi-square test of independence.

   H_0: Worker type and future plan are independent.
   H_a: Worker type and future plan are not independent.

   Observed and marginal counts:

75	24	99
43	23	66
118	47	165

   $\widehat{n_{ij}} = (n_{i\cdot})(n_{\cdot j})/n$
   $\widehat{n_{11}} = (99)(118)/165 = 70.8 \quad \widehat{n_{12}} = (99)(47)/165 = 28.2$
   $\widehat{n_{21}} = (66)(118)/165 = 47.2 \quad \widehat{n_{22}} = (66)(47)/165 = 18.8$

   $$h^* = \sum_i \sum_j \frac{(n_{ij} - \widehat{n_{ij}})^2}{\widehat{n_{ij}}}$$
   $$= \frac{(75-70.8)^2}{70.8} + \frac{(24-28.2)^2}{28.2} + \frac{(43-47.2)^2}{47.2} + \frac{(23-18.8)^2}{18.8} = 2.187$$

   $h_{.15,\,(2-1)(2-1)} = 2.072 < h^* = 2.187 < h_{.10,\,(2-1)(2-1)} = 2.706$
   $.10 < p\text{-value} < .15$

   Retain H_0 at $\alpha = .01$.
   (Insufficient evidence to conclude that future plan depends on worker type.)

9. Independent-samples t test by SPSS.

   $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 > 0$ (1 = "morning"; 2 = "afternoon")
   $t^* = 2.183$ (equal variances not assumed)
   one-sided p-value $= .047 \div 2 = .0235$
   Reject H_0 at $\alpha = .05$.
   The mean score for the morning group is significantly higher than the mean score for the afternoon group.