Math 220 : Elementary Statistics — Kane Nashimoto

Exam 3 Solutions

Spring 2009

1. Inference about mean.

   $H_0\colon \mu = 120$ vs. $H_a\colon \mu > 120$

   $t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{126.4 - 120}{16.2/\sqrt{60}} = 3.060$

   $t_{.0025,\,60-1} = 2.916 < t^* = 3.060 < t_{.001,\,60-1} = 3.234$

   $.001 < p\text{-value} < .0025$

   Reject $H_0$ at $\alpha = .01$. ($\mu > 120$)

2. Sampling distribution of mean.

   Note : $\bar{X}$, mean number of work hours, is approximately normal by the central limit theorem with $\mu_{\bar{X}} = \mu = 7.6$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 0.3/\sqrt{40}$.

   $P(\bar{X} > 7.5) = P\left(\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \dfrac{7.5 - 7.6}{0.3/\sqrt{40}}\right) = P(Z > -2.11)$

   $= 1 - P(Z \le -2.11) = 1 - .0174 = .9826$

3. Inference about proportion.

   (a) Use $\hat{p} = .5$.

   $n = \dfrac{\hat{p}(1-\hat{p})\, z^2_{.05/2}}{m^2} = \dfrac{.5(1-.5)(1.960)^2}{(.04)^2} = 600.25 \;\Rightarrow\; 601$

   (b) Use $\hat{p} = .30$.

   $n = \dfrac{\hat{p}(1-\hat{p})\, z^2_{.05/2}}{m^2} = \dfrac{.30(1-.30)(1.960)^2}{(.04)^2} = 504.21 \;\Rightarrow\; 505$

4. Inference about proportion.

   $H_0\colon p = .70$ vs. $H_a\colon p \ne .70$

   $z^* = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \dfrac{.68 - .70}{\sqrt{(.70)(1-.70)/1000}} = -1.380$

   $p\text{-value} = 2P(Z \le -1.38) = 2(.0838) = .1676$

   Retain $H_0$ at $\alpha = .10$. ($p \approx .70$)

5. Inference about mean.

   (a) $\bar{x} \pm t_{.05/2,\,35-1}\dfrac{s}{\sqrt{n}}$ ; $287.4 \pm 2.032\dfrac{70.1}{\sqrt{35}}$ ; $287.4 \pm 24.077$ ; $(263.323\,,\ 311.477)$

   (b) The confidence interval lies entirely right of 250. Therefore, the claim is supported.

   (c) $n = \dfrac{4s^2}{m^2} = \dfrac{4(70.1)^2}{(20)^2} = 49.14 \;\Rightarrow\; 50$