Math 220 : Elementary Statistics — Kane Nashimoto

Final Exam Solutions

Fall 2008

1. Binomial random variable.

   (a) $P(X = 5) = P(X \leq 5) - P(X \leq 4) = .8338 - .6331 = .2007$

   (b) $P(X > 2) = 1 - P(X \leq 2) = 1 - .5256 = .4744$

   (c) $\mu = np = (10)(.30) = 3.0$

2. Skewness. Standardized score.

   (a) The distribution of the data is negatively skewed (mean < median).

   (b) $\mu_{\text{new}} = 71.3$ (10 points higher)
   $\sigma_{\text{new}} = 14.1$ (no change)

3. Inference about mean.
   $H_0 : \mu = 13.50$ vs. $H_a : \mu \neq 13.50$
   $$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{13.52 - 13.50}{0.08/\sqrt{50}} = 1.768$$
   $t_{.05,\,50-1} = 1.677 < |t^*| = 1.768 < t_{.025,\,50-1} = 2.010$
   $.05 < p\text{-value} < .10$
   Retain H_0 at $\alpha = .01$. ($\mu \approx 13.50$)

4. Inference about proportion.

   (a) $\hat{p} = 229/638 = .359$
   $$\hat{p} \pm z_{.01/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\,;\ .359 \pm 2.576\sqrt{\frac{(.359)(1-.359)}{638}}\,;\ .359 \pm .049\,;\ (.310\,,\ .408)$$

   (b) Use $\hat{p} = .359$.
   $$n = \frac{\hat{p}(1-\hat{p})\,z^2_{.05/2}}{m^2} = \frac{.359(1-.359)(1.960)^2}{(.04)^2} = 552.52 \ \Rightarrow\ 553$$

5. Probability.
   A: {Have exceptionally high GPA}
   B: {Plan on going to graduate school}
   Note : $P(A) = .45\,;\ P(B) = .65\,;\ P(A \cap B) = .35$

   (a) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.35}{.65} = .538$

   (b) $P(A \cap B) = .35$ but $P(A)P(B) = (.45)(.65) = .293$
   $\Rightarrow$ Two events are not independent.

   (c) $P(\text{at least one graduate school}) = 1 - P(\text{no graduate school}) = 1 - (.35)^3 = .957$

6. Sampling distribution of mean.

   Note: X, price of steak cut, is normal with $\mu = 9.50$ and $\sigma = 0.35$. Thus, $\bar{X}$, mean price, is normal with $\mu_{\bar{X}} = \mu = 9.50$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 0.35/\sqrt{20}$.

   (a) $P(X > 9.75) = P\left(\dfrac{X-\mu}{\sigma} > \dfrac{9.75-9.50}{0.35}\right) = P(Z > 0.71)$
   $= 1 - P(Z \le 0.71) = 1 - .7611 = .2389$

   (b) $P(\bar{X} < 9.40) = P\left(\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \dfrac{9.40-9.50}{0.35/\sqrt{20}}\right) = P(Z < -1.28) = .1003$

7. Chi-square test of homogeneity.

   H_0: Relative proportions of status are homogeneous at two schools.
   H_a: Relative proportions of status are not homogeneous at two schools.

   Observed and marginal counts:

941	9	950
859	21	880
1800	30	1830

   $\widehat{n_{ij}} = (n_{i\cdot})(n_{\cdot j})/n$
   $\widehat{n_{11}} = (950)(1800)/1830 = 934.426$ $\quad \widehat{n_{12}} = (950)(30)/1830 = 15.574$
   $\widehat{n_{21}} = (880)(1800)/1830 = 865.574$ $\quad \widehat{n_{22}} = (880)(30)/1830 = 14.426$

   $$h^* = \sum_i \sum_j \frac{(n_{ij} - \widehat{n_{ij}})^2}{\widehat{n_{ij}}}$$
   $$= \frac{(941-934.426)^2}{934.426} + \frac{(9-15.574)^2}{15.574} + \frac{(859-865.574)^2}{865.574} + \frac{(21-14.426)^2}{14.426} = 5.867$$

   $h_{.025,\,(2-1)(2-1)} = 5.024 < h^* = 5.867 < h_{.01,\,(2-1)(2-1)} = 6.635$
   $.01 < p\text{-value} < .025$

   Reject H_0 at $\alpha = .05$.
   (Proportion of non-traditional students is significantly higher at school B.)

8. Experimental design.

   An example of a study in which two explanatory variables both affect the response variable and, hence, the true cause of the response is unclear.

9. Paired-samples t test by SPSS.

   $H_0: \mu_\delta = 0$ vs. $H_a: \mu_\delta \neq 0$ ("father's" minus "mother's")
   $t^* = -0.377$
   two-sided p-value $= .716$
   Retain H_0 at $\alpha = .05$.
   Insufficient evidence to conclude that fathers and mothers have had different amounts of education.