Math 220 : Elementary Statistics — Kane Nashimoto
Exam 3 Solutions
Fall 2008

1. Inference about proportion.

   $H_0\colon p = .75$ vs. $H_a\colon p < .75$

   $$z^* = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{.72 - .75}{\sqrt{(.75)(1-.75)/500}} = -1.549$$

   p-value $= P(Z \le -1.55) = .0606$

   Retain H_0 at $\alpha = .01$. ($p \approx .75$)

2. Sampling distribution of mean.

   (a) $\bar{X}$, number of hours working outside, is approximately normal by the central limit theorem with $\mu_{\bar{X}} = \mu = 5.9$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 0.6/\sqrt{40}$.

   (b) $P(\bar{X} < 6.0) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{6.0 - 5.9}{0.6/\sqrt{40}}\right) = P(Z < 1.05) = .8531$

3. Inference about proportion.

   (a) Use $\hat{p} = .5$.

   $$n = \frac{\hat{p}(1-\hat{p})\, z^2_{.10/2}}{m^2} = \frac{.5(1-.5)(1.645)^2}{(.04)^2} = 422.82 \;\Rightarrow\; 423$$

   (b) $\hat{p} = 268/390 = .687$

   $$\hat{p} \pm z_{.10/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\,;\; .687 \pm 1.645\sqrt{\frac{(.687)(1-.687)}{390}}\,;\; .687 \pm .039\,;\; (.648\,,\, .726)$$

4. Inference about mean.

   $H_0\colon \mu = 750$ vs. $H_a\colon \mu \ne 750$

   $$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{763 - 750}{29/\sqrt{50}} = 3.170$$

   $t_{.0025,\,50-1} = 2.940 < |t^*| = 3.170 < t_{.001,\,50-1} = 3.265$

   $.002 < p$-value $< .005$

   Reject H_0 at $\alpha = .01$. ($\mu > 750$)

5. Inference about mean.

   (a) $\bar{x} \pm t_{.05/2,\,70-1}\frac{s}{\sqrt{n}}\,;\; 51250 \pm 1.995\frac{10800}{\sqrt{70}}\,;\; 51250 \pm 2575.2\,;\; (48674.8\,,\, 53825.2)$

   (b) No, the true mean can be as low as \$48,675.

   (c) $n = \frac{4s^2}{m^2} = \frac{4(10800)^2}{(2000)^2} = 116.64 \;\Rightarrow\; 117$