Math 220 : Elementary Statistics — Kane Nashimoto
Final Exam Solutions
Spring 2008

1. Binomial random variable.

   Note : X, number of trucks, is binomial with $n = 20$ and $p = .30$.

   (a) $\mu = np = (20)(.30) = 6.0$
   (b) $P(X < 5) = P(X \leq 4) = .2375$
   (c) $P(X > 10) = 1 - P(X \leq 10) = 1 - .9829 = .0171$

2. Inference about mean.

   (a) $\bar{x} \pm t_{.01/2,\,30-1}\dfrac{s}{\sqrt{n}}$; $107 \pm 2.756\dfrac{28}{\sqrt{30}}$; 107 ± 14.089 ; $(92.911\,,\ 121.089)$
   (b) $n = \dfrac{4s^2}{m^2} = \dfrac{4(28)^2}{(10)^2} = 31.36 \;\Rightarrow\; 32$

3. Chi-square test of independence.

   H_0 : type and major are independent.
   H_a : type and major are not independent.

   Observed and marginal counts :

37	44	81
67	52	119
104	96	200

   $\widehat{n_{ij}} = (n_{i\cdot})(n_{\cdot j})/n$
   $\widehat{n_{11}} = (81)(104)/200 = 42.12 \quad \widehat{n_{12}} = (81)(96)/200 = 38.88$
   $\widehat{n_{21}} = (119)(104)/200 = 61.88 \quad \widehat{n_{22}} = (119)(96)/200 = 57.12$

   $$h^* = \sum_i \sum_j \frac{(n_{ij} - \widehat{n_{ij}})^2}{\widehat{n_{ij}}} = \frac{(37-42.12)^2}{42.12} + \frac{(44-38.88)^2}{38.88} + \frac{(67-61.88)^2}{61.88} + \frac{(52-57.12)^2}{57.12} = 2.179$$

   $h_{.15,\,(2-1)(2-1)} = 2.072 < h^* = 2.179 < h_{.10,\,(2-1)(2-1)} = 2.706$
   $.10 < p\text{-value} < .15$
   Retain H_0 at $\alpha = .05$.
   (Insufficient evidence to conclude that type and major are dependent.)

4. Cautions in analyzing association.

   An example of a study in which an unobserved variable influences the association between the variables of primary interest (e. g., high heat increases both consumption of ice cream and number of drowning incidents).

5. Probability.

   A: {woman adviser is assigned}
   B: {fulltime faculty adviser is assigned}
   Note: $P(A) = .35$; $P(B) = .75$; $P(A \cap B) = .20$

   (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .35 + .75 - .20 = .90$

   (b) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.20}{.35} = .571$

   (c) $P(A \cap B) = .20$ but $P(A)P(B) = (.35)(.75) = .263$
   $\Rightarrow$ Two events are not independent.

6. Numerical summary of data.

   Note: $\sum x = 295$; $\sum x^2 = 9025$; $n = 10$

   (a) $m = \dfrac{30 + 30}{2} = 30.000$

   (b) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{295}{10} = 29.500$

   (c) $s = +\sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n-1}} = +\sqrt{\dfrac{9025 - (295)^2/10}{10-1}} = 5.986$

7. Inference about proportion.

   $H_0 \colon p = .40$ vs. $H_a \colon p < .40$
   $\hat{p} = 361/950 = .38$
   $$z^* = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{.38 - .40}{\sqrt{(.40)(1-.40)/950}} = -1.258$$
   p-value $= P(Z \le -1.26) = .1038$
   Retain H_0 at $\alpha = .01$. ($p \approx .40$)

8. Normal random variable.

   Note: X, price of gasoline, is normal with $\mu = 3.45$ and $\sigma = 0.10$.

   (a) $P(X > 3.60) = P\left(\dfrac{X - \mu}{\sigma} > \dfrac{3.60 - 3.45}{0.10}\right) = P(Z > 1.50)$
   $= 1 - P(Z \le 1.50) = 1 - .9332 = .0668$

   (b) $z = -1.28 \because P(Z \le -1.28) \approx .10$
   $z = \dfrac{x - \mu}{\sigma}$; $-1.28 = \dfrac{x - 3.45}{0.10}$; $x = 3.45 + (-1.28)(0.10) = 3.32$

9. Independent-samples t test by SPSS.

   $H_0 \colon \mu_1 - \mu_2 = 0$ vs. $H_a \colon \mu_1 - \mu_2 > 0$ (1 = "Type A"; 2 = "Other")
   $t^* = 1.865$ (equal variances assumed)
   one-sided p-value $= .076 \div 2 = .038$
   Reject H_0 at $\alpha = .05$.
   The mean systolic pressure for individuals with Type A personalities is significantly higher than that for the other individuals.