Math 220 : Elementary Statistics — Kane Nashimoto
Exam 3 Solutions
Spring 2008

1. Inference about mean.

   (a) $\bar{x} \pm t_{.05/2,\,20-1}\dfrac{s}{\sqrt{n}}$ ; $103.10 \pm 2.093\dfrac{2.02}{\sqrt{20}}$ ; $103.10 \pm 0.945$ ; $(102.155\,,\ 104.045)$

   (b) Yes. The 95% confidence interval in (a) lies entirely right of 100.

   (c) $n = \dfrac{4s^2}{m^2} = \dfrac{4(2.02)^2}{(0.5)^2} = 65.29 \ \Rightarrow\ 66$

2. Inference about proportion.

   $H_0\colon p = .30$ vs. $H_a\colon p < .30$

   $\hat{p} = 72/250 = .288$

   $z^* = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \dfrac{.288 - .30}{\sqrt{(.30)(1-.30)/250}} = -0.414$

   $p$-value $= P(Z \le -0.41) = .3409$

   Retain $H_0$ at $\alpha = .01$. ($p \approx .30$)

3. Inference about proportion.

   (a) $\hat{p} = 18/750 = .024$

   $\hat{p} \pm z_{.10/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ ; $.024 \pm 1.645\sqrt{\dfrac{(.024)(1-.024)}{750}}$ ; $.024 \pm .009$ ; $(.015\,,\ .033)$

   (b) Use $\hat{p} = .024$.

   $n = \dfrac{\hat{p}(1-\hat{p})\,z^2_{.05/2}}{m^2} = \dfrac{.024(1-.024)(1.960)^2}{(.01)^2} = 899.86 \ \Rightarrow\ 900$

4. Inference about mean.

   $H_0\colon \mu = 60.0$ vs. $H_a\colon \mu \ne 60.0$

   $t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{56.2 - 60.0}{10.3/\sqrt{30}} = -2.021$

   $t_{.05,\,30-1} = 1.699 < |t^*| = 2.021 < t_{.025,\,30-1} = 2.045$

   $.05 < p\text{-value} < .10$

   Reject $H_0$ at $\alpha = .10$. ($\mu < 60.0$)

5. Sampling distribution of mean.

   (a) $\bar{X}$, mean weight, is approximately normal by the central limit theorem with $\mu_{\bar{X}} = \mu = 5.08$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 0.06/\sqrt{30}$.

   (b) $P(\bar{X} > 5.05) = P\left(\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \dfrac{5.05 - 5.08}{0.06/\sqrt{30}}\right) = P(Z > -2.74)$

   $= 1 - P(Z \le -2.74) = 1 - .0031 = .9969$