Math 220 : Elementary Statistics — Kane Nashimoto
Final Exam Solutions
Summer 2007

1. Numerical and graphical summaries of data.

   Note : $\sum x = 237$; $\sum x^2 = 6373$; $n = 10$

   (a)

Stem	Leaf
0	7
1	48
2	0457
3	336

   (b) The distribution of the data is negatively skewed.

   (c) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{237}{10} = 23.700$

   $s = +\sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n-1}} = +\sqrt{\dfrac{6373 - (237)^2/10}{10-1}} = 9.166$

2. An example of a study in which two factors are confounded.

3. Inference about mean with σ unknown.

   (a) $\bar{x} \pm t_{.05/2,\,25-1}\dfrac{s}{\sqrt{n}}$; $4.22 \pm 2.064\dfrac{0.27}{\sqrt{25}}$; 4.22 ± 0.111 ; $(4.109\,,\ 4.331)$

   (b) $H_0\colon \mu = 4.20$ vs. $H_a\colon \mu > 4.20$

   $t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{4.22 - 4.20}{0.27/\sqrt{25}} = 0.370$

   Critical value : -1.711 $(t_{.05,\,25-1} = 1.711)$

   Retain H_0. $(\mu \approx 4.20)$

4. Binomial random variable.

   Note : X, number of fish turning right, is binomial with $n = 20$ and $\pi = .80$.

   (a) $P(X = 12) = P(X \le 12) - P(X \le 11) = .0321 - .0100 = .0221$

   (b) $P(X < 15) = P(X \le 14) = .1958$

5. Chi-square goodness-of-fit test.

   $H_0\colon \pi_1 = .20$; $\pi_2 = .40$; $\pi_3 = .40$

   $H_a\colon \pi_i \ne \pi_{0i}$ for some $1 \le i \le 3$

   $E_i = n\pi_{0i}$; $n = 870$

   $E_1 = (870)(.20) = 174.0$ $\quad E_2 = (870)(.40) = 348.0$ $\quad E_3 = (870)(.40) = 348.0$

   $h^* = \sum_i \dfrac{(O_i - E_i)^2}{E_i} = \dfrac{(183 - 174.0)^2}{174.0} + \dfrac{(327 - 348.0)^2}{348.0} + \dfrac{(360 - 348.0)^2}{348.0} = 2.147$

   Critical value : 4.605 $(h_{.10,\,3-1} = 4.605)$

   Retain H_0.

   (Insufficient evidence to refute the manager's claim.)

6. Normal random variable.

   Note: X, price of 12-pack soda, is normal with $\mu = 3.00$ and $\sigma = 0.25$.

   (a) $P(X > 3.15) = P\left(\frac{X-\mu}{\sigma} > \frac{3.15-3.00}{0.25}\right) = P(Z > 0.60)$
   $= 1 - P(Z \leq 0.60) = 1 - .7257 = .2743 \;\Rightarrow\; 27.43\%$

   (b) $z = -0.52 \;\because\; P(Z \leq -0.52) \approx .30$
   $z = \frac{x-\mu}{\sigma}\,;\; -0.52 = \frac{x-3.00}{0.25}\,;\; x = 3.00 + (-0.52)(0.25) = 2.87$

7. Probability.

   E: {Resident owns a domestic car}
   F: {Resident owns a four-door sedan}
   Note: $P(E) = .45\,;\; P(F) = .76\,;\; P(E \cap F) = .28$

   (a) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = .45 + .76 - .28 = .93$

   (b) $P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{.28}{.45} = .622$

   (c) $P(F_1 \cap F_2) = P(F_1)P(F_2) = (.76)^2 = .578$

8. Independent-samples t test by SPSS.

   $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 > 0$ (1 = "with notes"; 2 = "without notes")
   $t^* = 0.514$ (equal variances assumed)
   one-sided p-value $= .611 \div 2 = .306$ $(> \alpha = .05)$
   Retain H_0.
   Insufficient evidence to conclude that providing lecture notes leads to higher exam scores.