Math 220 : Elementary Statistics — Kane Nashimoto
Exam 3 Solutions
Summer 2007

1. Inference about proportion.

   (a) $H_0\colon \pi = .20$ vs. $H_a\colon \pi < .20$
   $p = 34/200 = .170$
   $$z^* = \frac{p - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} = \frac{.170 - .20}{\sqrt{(.20)(1-.20)/200}} = -1.061$$
   Critical value : -1.645 $(z_{.05} = 1.645)$
   Retain H_0. $(\pi \approx .20)$

   (b) Let $\pi = p$.
   $$n = \pi(1-\pi)\left(\frac{z_{.01/2}}{B}\right)^2 = (.17)(1-.17)\left(\frac{2.576}{.05}\right)^2 = 374.52 \Rightarrow 375$$

   (c) Let $\pi = .5$.
   $$n = \pi(1-\pi)\left(\frac{z_{.01/2}}{B}\right)^2 = (.5)(1-.5)\left(\frac{2.576}{.05}\right)^2 = 663.58 \Rightarrow 664$$

2. Sampling distribution.

   Note : $\bar{X}$, mean number of sick days used, is approximately normal by CLT.

   (a) $\mu_{\bar{X}} = \mu = 7.00$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 2/\sqrt{100} = 0.20$

   (b) $P(\bar{X} < 6.5) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{6.5 - 7.00}{2/\sqrt{100}}\right) = P(Z < -2.50) = .0062$

3. Inference about mean with σ unknown.

   (a) $\bar{x} \pm t_{.02/2,\,19-1}\frac{s}{\sqrt{n}}$; $4.07 \pm 2.552\frac{0.16}{\sqrt{19}}$; 4.07 ± 0.094 ; $(3.976\,,\ 4.164)$

   (b) The confidence interval in (a) contains 4.0, indicating that 4.0 is a plausible value of the true mean weight. There is insufficient evidence to argue that the label is inaccurate.

4. Inference about mean with σ known.

   (a) $H_0\colon \mu = 120.0$ vs. $H_a\colon \mu \neq 120.0$
   $$z^* = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{123.7 - 120.0}{12.6/\sqrt{60}} = 2.275$$
   Critical values : ± 1.960 $(z_{.05/2} = 1.960)$
   Reject H_0. $(\mu > 120.0)$

   (b) p-value $= 2P(Z \geq 2.25) = 2(1 - P(Z < 2.25)) = 2(1 - .9878) = 2(.0122) = .0244$

   (c) Reject H_0. If a test rejects H_0 at $\alpha = .05$, it will reject H_0 at any $\alpha > .05$.