Math 220 : Elementary Statistics — Kane Nashimoto
Final Exam Solutions
Spring 2007

1. Binomial random variable.

   Note : X, number of bulbs burned out, is binomial with $n = 20$ and $\pi = .30$.

   (a) $P(X = 7) = P(X \le 7) - P(X \le 6) = .7723 - .6080 = .1643$
   (b) $P(X > 10) = 1 - P(X \le 10) = 1 - .9829 = .0171$

2. Normal random variable.

   Note : X, commuting time, is normal with $\mu = 20$ and $\sigma = 4$.

   (a) $P(X > 25) = P\left(\dfrac{X-\mu}{\sigma} > \dfrac{25-20}{4}\right) = P(Z > 1.25)$
   $= 1 - P(Z \le 1.25) = 1 - .8944 = .1056$
   (b) $P(X \le 30) = P\left(\dfrac{X-\mu}{\sigma} \le \dfrac{30-20}{4}\right) = P(Z \le 2.50) = .9938$
   (c) $z = 0.44 \ \because\ P(Z \le 0.44) \approx .67$
   $z = \dfrac{x-\mu}{\sigma}\,;\ 0.44 = \dfrac{x-20}{4}\,;\ x = 20 + (0.44)(4) = 21.76$

3. Inference about mean with σ unknown.

   (a) $H_0\colon \mu = 3.00$ vs. $H_a\colon \mu > 3.00$
   $t^* = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}} = \dfrac{3.15-3.00}{0.50/\sqrt{22}} = 1.407$
   Critical value : $+1.721$ $(t_{.05,\,22-1} = 1.721)$
   Retain H_0. $(\mu \approx 3.00)$
   (b) Type II error (concluding that the mean does not exceed 3.00 ppm when in fact it does) is more serious because the workers may be exposed to contaminated air. Use a larger α to reduce the risk of committing a Type error.

4. Experimental design.

   (a) $\dfrac{482}{1000} = .482$
   (b) The result can be generalized to the population of the residents in Henrico County.

5. Numerical summary of data.

   Note : $\sum x = 26\,;\ \sum x^2 = 226\,;\ n = 5$

   (a) $m = 4$
   (b) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{26}{5} = 5.200$
   $s = +\sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n-1}} = +\sqrt{\dfrac{226-(26)^2/5}{5-1}} = 4.764$

6. Chi-square test of independence.

   H_0: Helment use and type of injury are independent.
   H_a: Helment use and type of injury are not independent.

   Observed and marginal counts:

26	44	70
79	51	130
105	95	200

   $E_{ij} = (O_{i\cdot})(O_{\cdot j})/n$
   $E_{11} = (70)(105)/200 = 36.75 \quad E_{12} = (70)(95)/200 = 33.25$
   $E_{21} = (130)(105)/200 = 68.25 \quad E_{22} = (130)(95)/200 = 61.75$

   $$h^* = \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$
   $$= \frac{(26-36.75)^2}{36.75} + \frac{(44-33.25)^2}{33.25} + \frac{(79-68.25)^2}{68.25} + \frac{(51-61.75)^2}{61.75} = 10.185$$

   Critical value: 6.635 ($h_{.01,\,(2-1)(2-1)} = 6.635$)
   Reject H_0.
   (Facial injury is less likely with helmets.)

7. Probability.

   E: {flower is yellow}
   F: {flower has hairy leaves}

   Note: $P(E) = .40$; $P(F) = .70$; $P(E \cap F) = .30$

   (a) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = .40 + .70 - .30 = .800$

   (b) $P(F|E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{.30}{.40} = .750$
   $\Rightarrow$ 75%

   (c) $P(E \cap F) = .30$ but $P(E)P(F) = (.40)(.70) = .280$
   $\Rightarrow$ Two events are not independent.

8. Paired-samples t test by SPSS.

   $H_0: \mu_\delta = 0$ vs. $H_a: \mu_\delta > 0$ ("before treatment" minus "after treatment")
   $t^* = 1.186$
   one-sided p-value $= .280 \div 2 = .140$ ($> \alpha = .05$)
   Retain H_0.
   Insufficient evidence to conclude that the sleeping pill is effective.