

Name: \_\_\_\_\_

Directions: This exam contains eight problems worth a total of 100 points. For each computational problem, you must first write the formula to be used and present all your subsequent work in order to receive full or partial credit. Circle your final answers.

1. A certain brand of a light bulb burs out with probability .30 before 1,000 hours of use. A company just installed 20 of these light bulbs in a conference room. Assume that the light bulbs burn out, if they do, independently.
  - (a) Compute the probability that exactly 7 of the 20 light bulbs will burn out before 1,000 hours of use. (5 pts.)
  
  
  
  
  
  
  
  
  
  
  - (b) Compute the probability that more than 10 of the 20 light bulbs will burn out before 1,000 hours of use. (5 pts.)
  
  
  
  
  
  
  
  
  
  
2. The time it takes you to drive to work is approximately normally distributed with a mean of 20 minutes and a standard deviation of 4 minutes.
  - (a) Calculate the probability that your commuting time will be more than 25 minutes next Wednesday. (5 pts.)
  
  
  
  
  
  
  
  
  
  
  - (b) If you leave your house at 7:30 a.m. next Wednesday, what is the probability that you will arrive at your work place by 8:00 a.m.? (5 pts.)
  
  
  
  
  
  
  
  
  
  
- (c) You will arrive at your work place within  $x$  minutes 67% of the time. Find the value of  $x$ . (5 pts.)

3. The EPA sets an airborne limit of 5.00 ppm on vinyl chloride, a colorless gas used to make plastics and adhesives. A plastics manufacturer must halt production when the mean amount of vinyl chloride in the air exceeds 3.00 ppm. A random sample of 22 air specimens produced a mean of 3.15 ppm with a standard deviation of 0.50 ppm. Assume that the distribution of the amount of vinyl chloride is approximately normal.

- (a) Conduct a test to determine whether the production process should be halted. Use  $\alpha = .05$ . (10 pts.)

$H_0$ : \_\_\_\_\_ vs.  $H_a$ : \_\_\_\_\_

Compute the test statistic and define the rejection rule.

Should the null hypothesis be rejected? Circle one.      Yes      No

- (b) If you were the factory manager, would you use a larger or smaller  $\alpha$  for the test in (a)? Give a brief explanation by considering the relative seriousness of the Type I and Type II errors in this context. (5 pts.)

4. A telephone survey was conducted to estimate the percentage of the people who purchase common goods (e. g., clothes, shoes, gifts) regularly over the Internet. One thousand people were randomly selected from the Henrico County telephone directory and were contacted. Four hundred eighty-two chose to participate in the survey. Of those who participated in the survey, 27% said that they shopped regularly over the Internet.

- (a) Compute the response rate of the survey. (5 pts.)

- (b) To what population can the result of the survey (approximately 27% shop regularly over the Internet) be generalized? (5 pts.)

5. Featured below are numbers of drinks consumed by a sample of five students for a period of one week.

$$X: \{5, 4, 0, 4, 13\}$$

(a) Compute the sample median. (5 pts.)

(b) Compute the sample mean and the sample standard deviation. (10 pts.)

6. A city government is interested in investigating whether bicycle helmets are truly effective. From a record, a random sample of 200 bicycle-crash cases was obtained, and the persons involved in the accidents were cross-classified according to whether they were wearing a helmet or not at the time of accident and whether the injuries were facial or non-facial. Conduct a test to find an answer to the city government's question. Use  $\alpha = .01$ . (10 pts.)

| Helmet   | Injury |            |
|----------|--------|------------|
|          | Facial | Non-facial |
| Worn     | 26     | 44         |
| Not worn | 79     | 51         |

$H_0$ :

$H_a$ :

Compute the test statistic and define the rejection rule.

Should the null hypothesis be rejected? Circle one.      Yes      No

7. For a particular species of a wild flower, 40% of the flowers are yellow (rest of the flowers are white), 70% have hairy leaves (others have smooth leaves), and 30% are yellow and have hairy leaves.

(a) If you pick one flower of this species at random, what is the probability that the flower will be yellow, have hairy leaves, or both? (5 pts.)

(b) What percentage of yellow flowers of this species have hairy leaves? (5 pts.)

(c) Are the flower being yellow and having hairy leaves independent? Give a mathematical justification. (5 pts.)

8. The effectiveness of a new sleeping pill was investigated. A randomly selected seven adults who have had sleeping problems were asked to record the number of days in which they were unable to sleep during a 20-day period before and after the treatment. If the sleeping pill is effective, there should be a decline in number of sleepless nights after the treatment.

The obtained data were analyzed using SPSS. The results of the analysis are shown on the following page.

(over)

Figure 1. SPSS output for Problem 8.

| Paired Samples Statistics |                  |       |   |                |                 |
|---------------------------|------------------|-------|---|----------------|-----------------|
|                           |                  | Mean  | N | Std. Deviation | Std. Error Mean |
| Pair 1                    | Before treatment | 11.57 | 7 | 4.685          | 1.771           |
|                           | After treatment  | 10.00 | 7 | 2.708          | 1.024           |

  

| Paired Samples Correlations |                                    |   |             |      |
|-----------------------------|------------------------------------|---|-------------|------|
|                             |                                    | N | Correlation | Sig. |
| Pair 1                      | Before treatment & After treatment | 7 | .670        | .100 |

  

| Paired Samples Test |                                    |                    |                |                 |   |       |       |    |                 |
|---------------------|------------------------------------|--------------------|----------------|-----------------|---|-------|-------|----|-----------------|
|                     |                                    | Paired Differences |                |                 |   |       | t     | df | Sig. (2-tailed) |
|                     |                                    | Mean               | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference |       |       |    |                 |
|                     |                                    |                    |                |                 | Lower                                     | Upper |       |    |                 |
| Pair 1              | Before treatment - After treatment | 1.571              | 3.505          | 1.325           | -1.670                                    | 4.813 | 1.186 | 6  | .280            |

For this analysis, provide a summary of the results. If necessary, use a significance level of .05. (10 pts.)

- State the null and alternative hypotheses.
- Report the test statistic.
- Report the observed significance level.
- State the decision (reject or retain  $H_0$ ).
- Interpret the results in the context of the problem.