Math 220 : Elementary Statistics Kane Nashimoto
Final Exam Solutions
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1. Inference about mean with σ unknown.

   $H_0 : \mu = 1.00$ vs. $H_a : \mu \neq 1.00$

   $t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{0.99 - 1.00}{0.03/\sqrt{30}} = -1.826$

   Critical values : ± 2.045 $(t_{.05/2,\,30-1} = 2.045)$

   Retain H_0. $(\mu \approx 1.00)$

2. Binomial random variable.

   Note : X, number of customers turning right, is binomial with $n = 25$ and $\pi = .90$.

   (a) $P(X = 23) = P(X \leq 23) - P(X \leq 22) = .7288 - .4629 = .2659$

   (b) $P(15 \leq X \leq 20) = P(X \leq 20) - P(X \leq 14) = .0980 - .0000 = .0980$

3. Sampling distribution.

   Note : X, number of pages printed per month, is normal with $\mu = 12000$ and $\sigma = 900$. Therefore, $\bar{X}$, mean number of pages printed per month, is normal with $\mu_{\bar{X}} = \mu = 12000$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 900/\sqrt{12}$.

   (a) $P(X < 10000) = P\left(\dfrac{X - \mu}{\sigma} < \dfrac{10000 - 12000}{900}\right) = P(Z < -2.22) = .0132$

   (b) Total number of pages printed per year exceeds 153,000 whenever the mean number of pages printed per month, $\bar{X}$, exceeds 153,000 $\div$ 12 = 12,750.
   $\Rightarrow$ Any $\bar{x} > 12750$.

   (c) $P(\bar{X} > 12750) = P\left(\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \dfrac{12750 - 12000}{900/\sqrt{12}}\right) = P(Z > 2.89)$

   $= 1 - P(Z \leq 2.89) = 1 - .9981 = .0019$

4. Inference about proportion.

   (a) $p = 108/237 = .456$

   $p \pm z_{.01/2}\sqrt{\dfrac{p(1-p)}{n}}$; $.456 \pm 2.576\sqrt{\dfrac{(.456)(1-.456)}{237}}$; $.456 \pm .083$; $(.373\,,\,.539)$

   (b) Let $\pi = p$.

   $n = \pi(1-\pi)\left(\dfrac{z_{.01/2}}{B}\right)^2 = (.456)(1-.456)\left(\dfrac{2.576}{.15/2}\right)^2 = 292.64 \;\Rightarrow\; 293$

5. Chi-square goodness-of-fit test.

   $H_0: \pi_1 = .40\,;\ \pi_2 = .50\,;\ \pi_3 = .10$
   $H_a: \pi_i \neq \pi_{0i}$ for some $1 \leq i \leq 3$
   $E_i = n\pi_{0i}\,;\ n = 310$
   $E_1 = (310)(.40) = 124.0 \quad E_2 = (310)(.50) = 155.0 \quad E_3 = (310)(.10) = 31.0$

   $$h^* = \sum_i \frac{(O_i - E_i)^2}{E_i} = \frac{(119 - 124.0)^2}{124.0} + \frac{(162 - 155.0)^2}{155.0} + \frac{(29 - 31.0)^2}{31.0} = 0.647$$

   Critical value: 5.991 $(h_{.05,\,3-1} = 5.991)$
   Retain H_0.
   (Insufficient evidence to conclude that the restaurant owner's belief is incorrect.)

6. Numerical summary of data.

   Note: $\sum x = 52\,;\ n = 10$

   (a) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{52}{10} = 5.200$

   $m = \dfrac{5+5}{2} = 5.000$

   (b) The distribution of the data is positively skewed (mean > median).

7. Probability.

   E: {student having checking account}
   F: {student having savings account}
   Note: $P(E) = .70\,;\ P(F) = .35\,;\ P(E \cap F) = .20$

   (a) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = .70 + .35 - .20 = .850$

   (b) $P(E|F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{.20}{.35} = .571$

   (c) $1 - P(E \cup F) = 1 - .850 = .150$

   (d) $P(E \cap F) = .20$ but $P(E)P(F) = (.70)(.35) = .245$
   $\Rightarrow$ Two events are not independent.

8. Independent-samples t test by SPSS.

   $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 \neq 0$ (1 = "Private"; 2 = "Public")
   $t^* = 1.806$ (equal variances not assumed)
   two-sided p-value $= .077$ $(< \alpha = .10)$
   Reject H_0.
   Students attending private colleges favor the governmental financial support significantly more than do students attending public universities.