Math 220: Elementary Statistics — Kane Nashimoto
Final Exam Solutions
Summer 2006

1. Bias in scientific studies.

   An example of a study in which the sample is not representative of the target population.

2. Chi-square test of homogeneity.

   (a) H_0: Relative proportions of trees are homogeneous at two sites.
   H_a: Relative proportions of trees are not homogeneous at two sites.

   Observed and marginal counts:

77	41	118
84	93	177
161	134	295

   $E_{ij} = (O_{i\cdot})(O_{\cdot j})/n$
   $E_{11} = (118)(161)/295 = 64.4 \quad E_{12} = (118)(134)/295 = 53.6$
   $E_{21} = (177)(161)/295 = 96.6 \quad E_{22} = (177)(134)/295 = 80.4$

   $$h^* = \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$
   $$= \frac{(77-64.4)^2}{64.4} + \frac{(41-53.6)^2}{53.6} + \frac{(84-96.6)^2}{96.6} + \frac{(93-80.4)^2}{80.4} = 9.045$$

   Critical value: 6.635 ($h_{.01,(2-1)(2-1)} = 6.635$)
   Reject H_0.
   (There are more coniferous trees in the north; there are more deciduous trees in the south.)

   (b) $E_{ij} > 5$ for all i, j
   $\Rightarrow$ Assumption is satisfied.

3. Binomial random variable.

   Note: X, number of black cars, is binomial with $n = 10$ and $\pi = .10$.

   (a) $P(X = 6) = P(X \leq 6) - P(X \leq 5) = 1.000 - .9999 = .0001$
   (b) $P(X \geq 1) = 1 - P(X = 0) = 1 - .3487 = .6513$

4. Inference about mean with σ unknown.

   (a) $H_0: \mu = 47$ vs. $H_a: \mu < 47$
   $$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{46.94 - 47}{0.15/\sqrt{20}} = -1.789$$
   Critical value: -1.729 ($t_{.05,\,20-1} = 1.729$)
   Reject H_0. ($\mu < 47$)

   (b) $\bar{x} \pm t_{.05/2,\,20-1} \dfrac{s}{\sqrt{n}}$; $46.94 \pm 2.093 \dfrac{0.15}{\sqrt{20}}$; 46.94 ± 0.070 ; $(46.870\,,\ 47.010)$

5. Sampling distribution.

   Note: $\bar{X}$, mean play time of songs, is approximately normal by CLT with $\mu_{\bar{X}} = \mu = 225$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 40/\sqrt{30}$.

   (a) Total play time exceeds 2 hours (120 minutes) whenever the mean play time, $\bar{X}$, exceeds $120 \div 30 = 4$ minutes (240 seconds). $\Rightarrow$ Any $\bar{x} > 240$.

   (b) $P(\bar{X} \geq 240) = P\left(\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq \dfrac{240 - 225}{40/\sqrt{30}}\right) = P(Z \geq 2.05)$

   $= 1 - P(Z < 2.05) = 1 - .9798 = .0202$

6. Probability.

   Option A:

   E: {selected resident is woman}
   F: {selected resident works fulltime}

   Note: $P(E) = .55$; $P(F) = .85$; $P(F|E) = .80$

   (a) $P(E \cap F) = P(E)P(F|E) = (.55)(.80) = .44$

   (b) $P(E|F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{.44}{.85} = .518$

   Option B:

   (a) 1/3.

   (b) Still 1/3. Alice already knew that at least one of Bob and Chris was not hired. Learning which person was not hired does not provide new information.

7. Graphical summary of data.

   Note: $\sum x = 226$; $n = 17$

   (a) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{226}{17} = 13.294$

   (b) $\bar{x}_{\text{new}} = 16.294$ (3 points higher)
   $s_{\text{new}} = 8.053$ (no change in variability)

8. Paired-samples t test by SPSS.

   $H_0: \mu_\delta = 0$ vs. $H_a: \mu_\delta \neq 0$ ("East side" minus "West side")
   $t^* = -2.357$
   two-sided p-value $= .057$ $(> \alpha = .05)$
   Retain H_0.
   Insufficient evidence to conclude that the mean numbers of cars parked are significantly different between the east-side and west-side lots.